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Modelling the Robot Through Robot Kinematics and Dynamics

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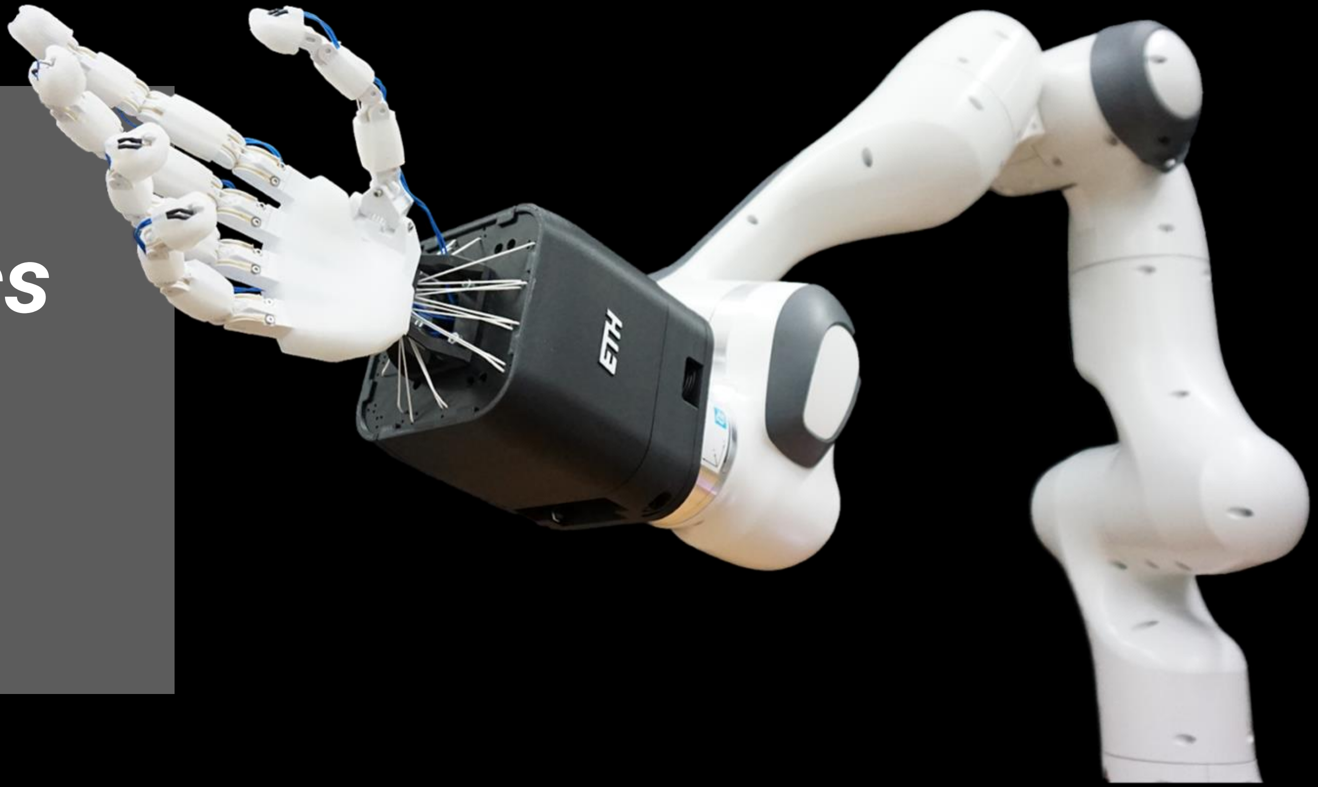


Faive Robotics

SoftRobotics Laboratory

Last Tutorial

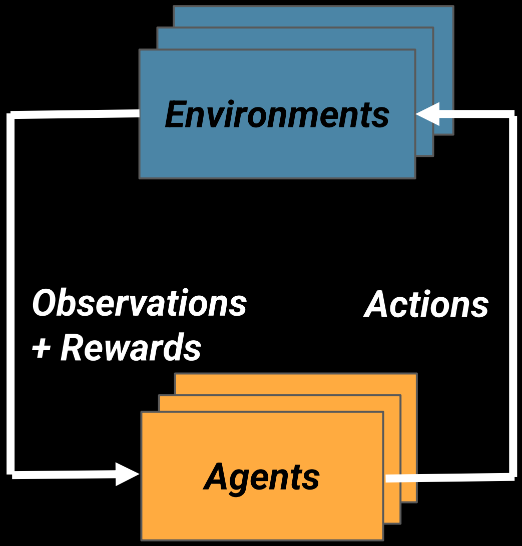
SOFA

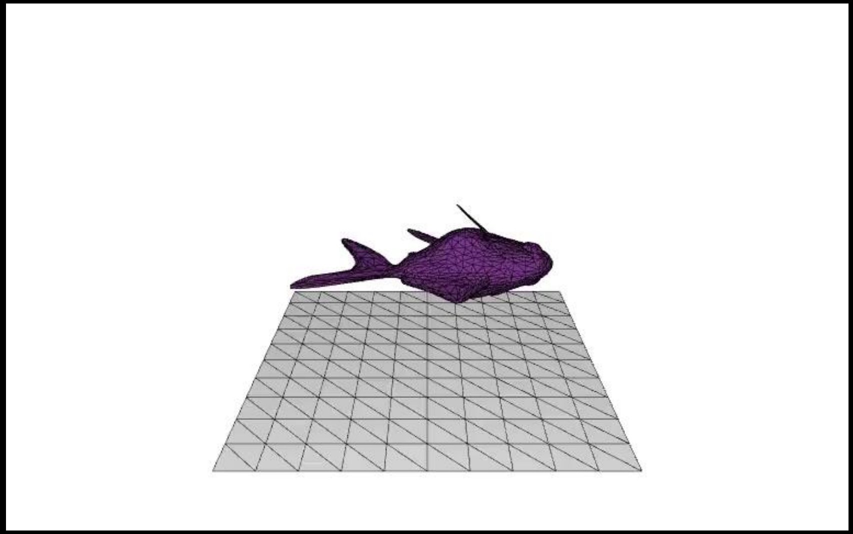
NVIDIA ISAAC

GAZEBO

MuJoCo

DRAKE



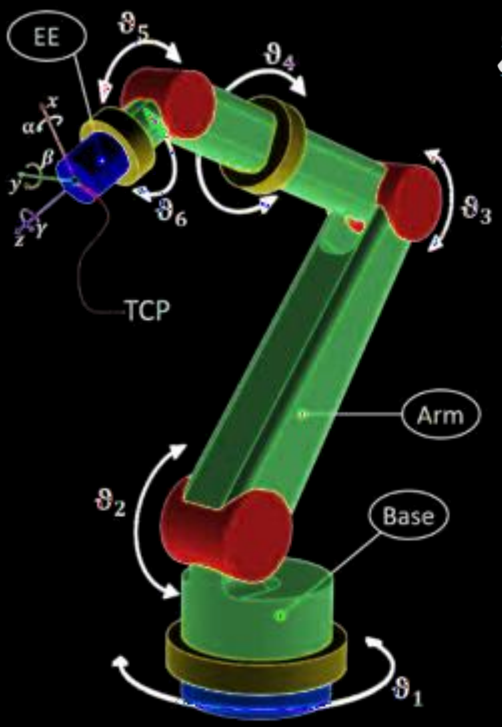


Last Tutorial

USA Toyz

How does it move?

Joint angles → end effector orientation?



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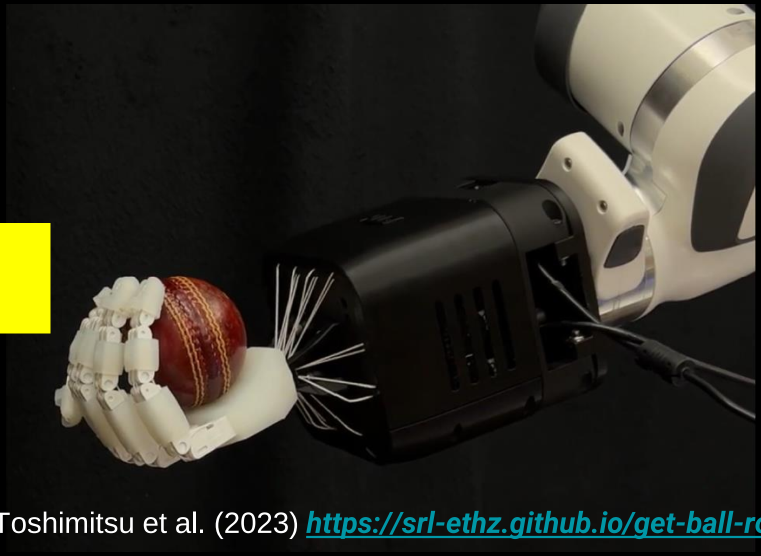
Robot Kinematics & Robot Dynamics

Robotis

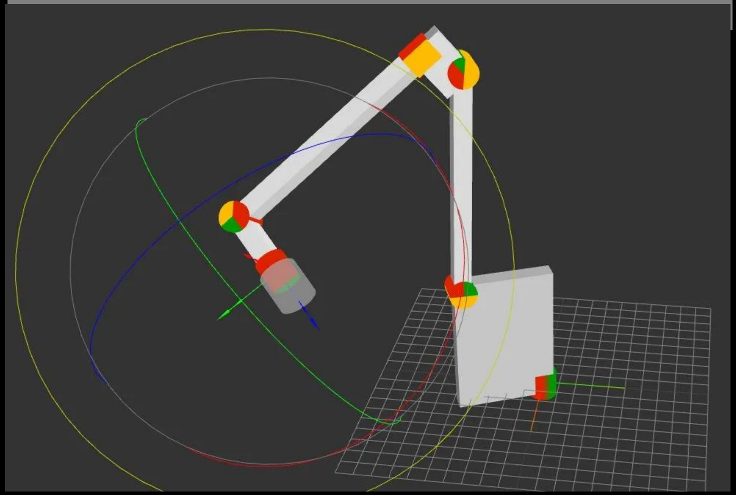
Motor ↔ Fingertips
Input? Force?



Toshimitsu et al. (2023) https://srl-ethz.github.io/get-ball-rolling/

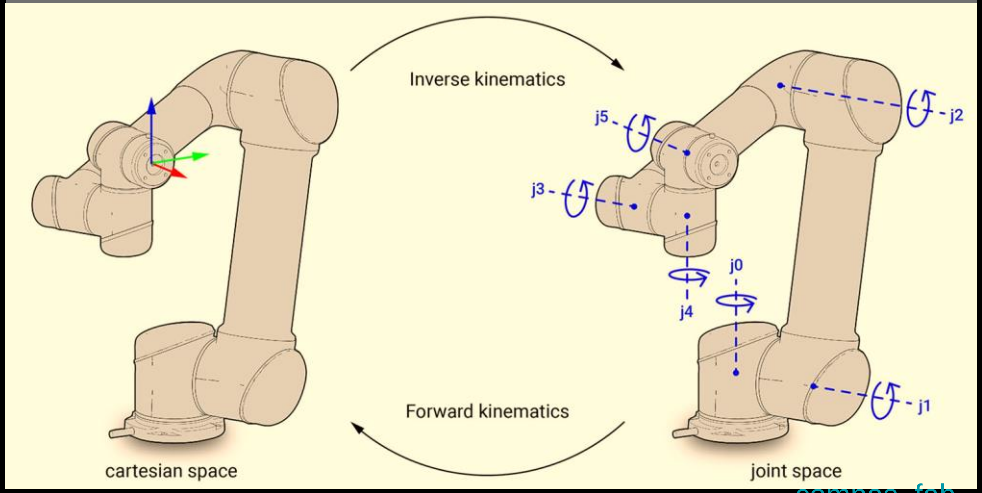
Plan for Today

1. Robot Kinematics and Dynamics



Marginally Clever Robots

2. Forward and Inverse Kinematics



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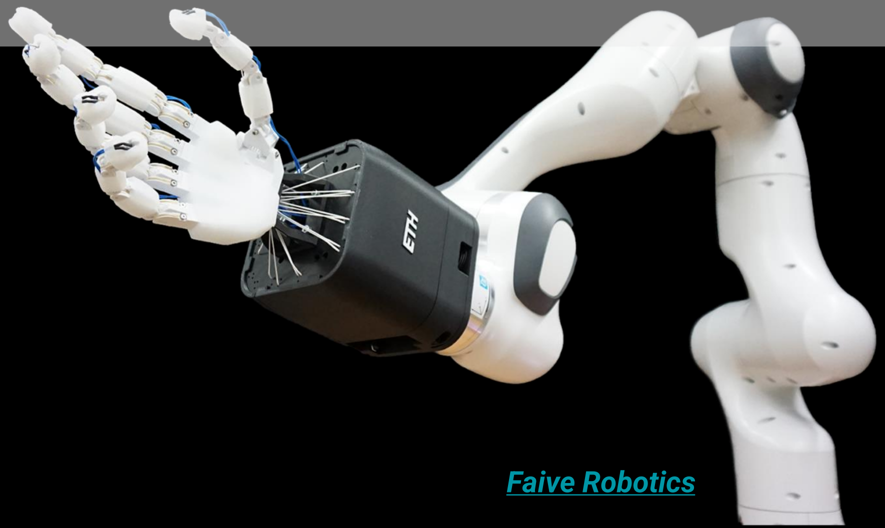
3. Kinematics and Dynamics for hand joints



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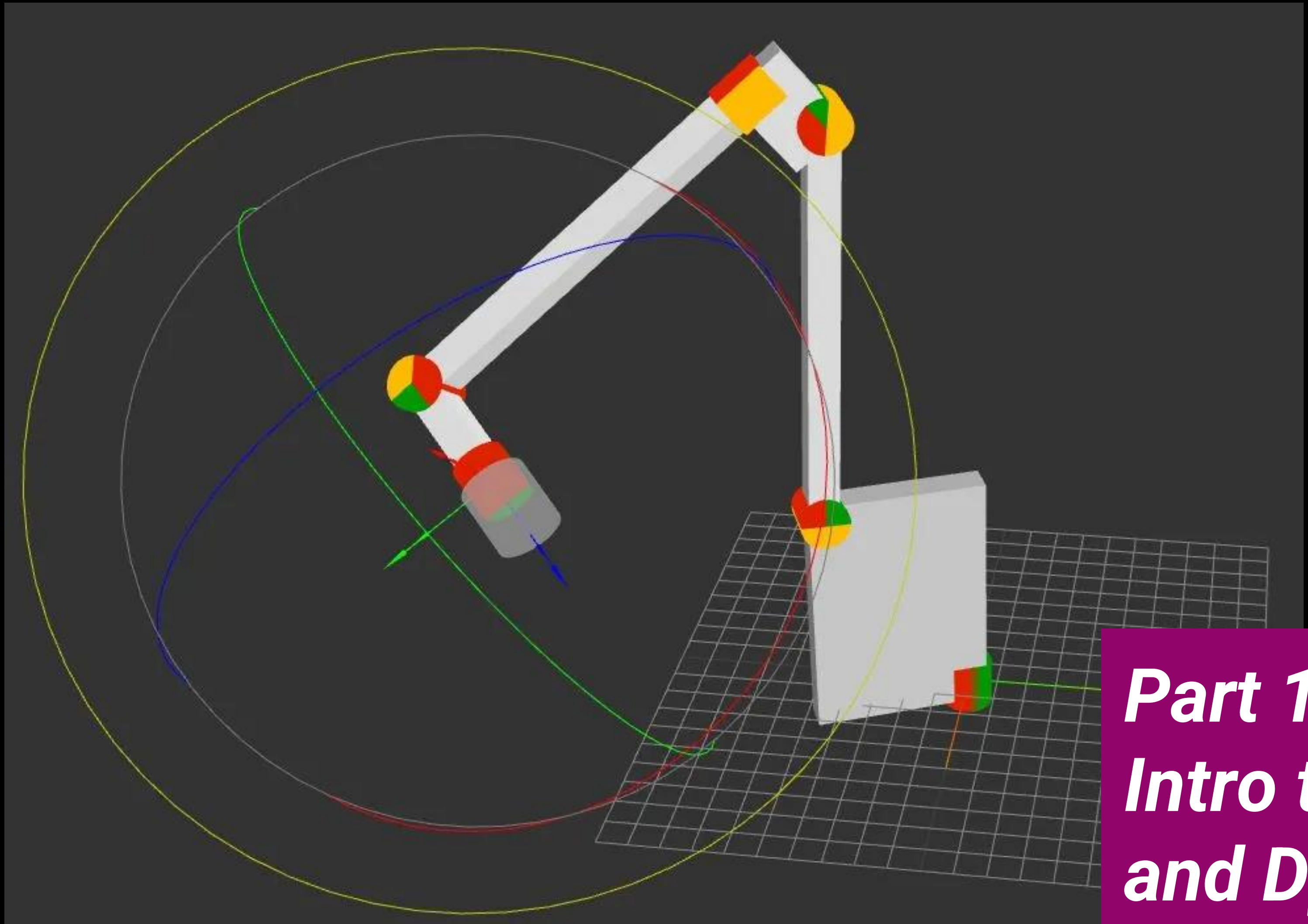

Marginally Clever Robots

Part 1: Intro to Robot Kinematics and Dynamics

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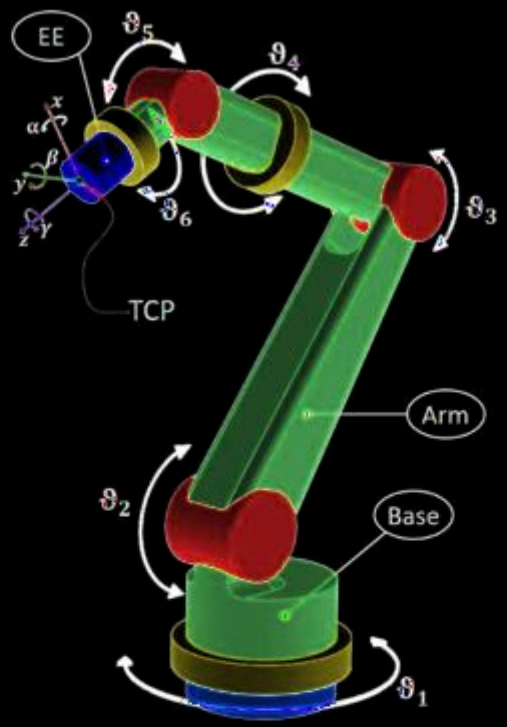
Robot Kinematics and Dynamics



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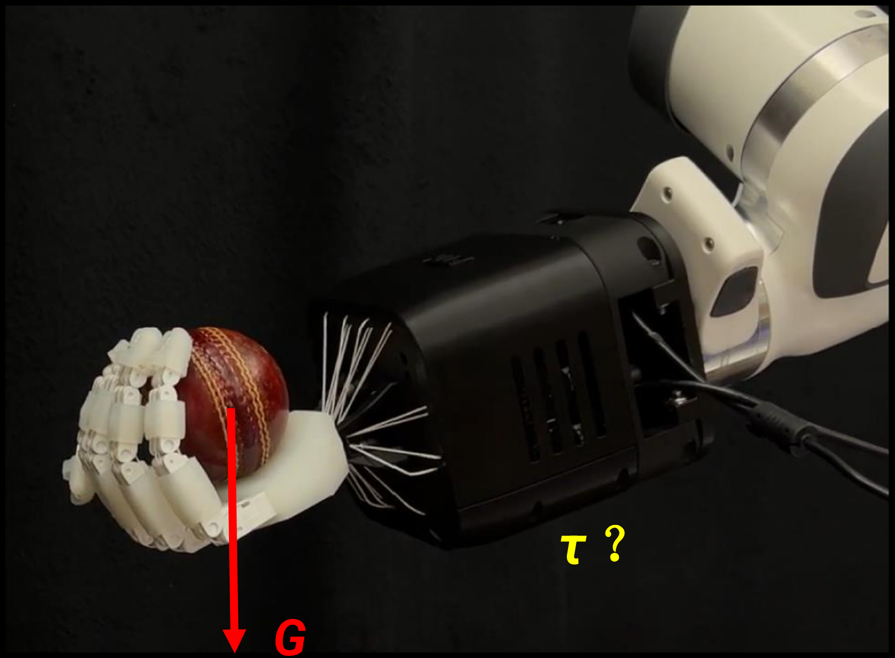
Kinematics



Toshimitsu et al. (2023) https://srl-ethz.github.io/get-ball-rolling/

Dynamics

Simulation
reaction to certain actuator commands

Control
invert of simulation, if I want to get somewhere, what command to give?

Design
how are the loads distributed

Optimization
what dimension should I have

Actuation
torque, speed, powder etc.

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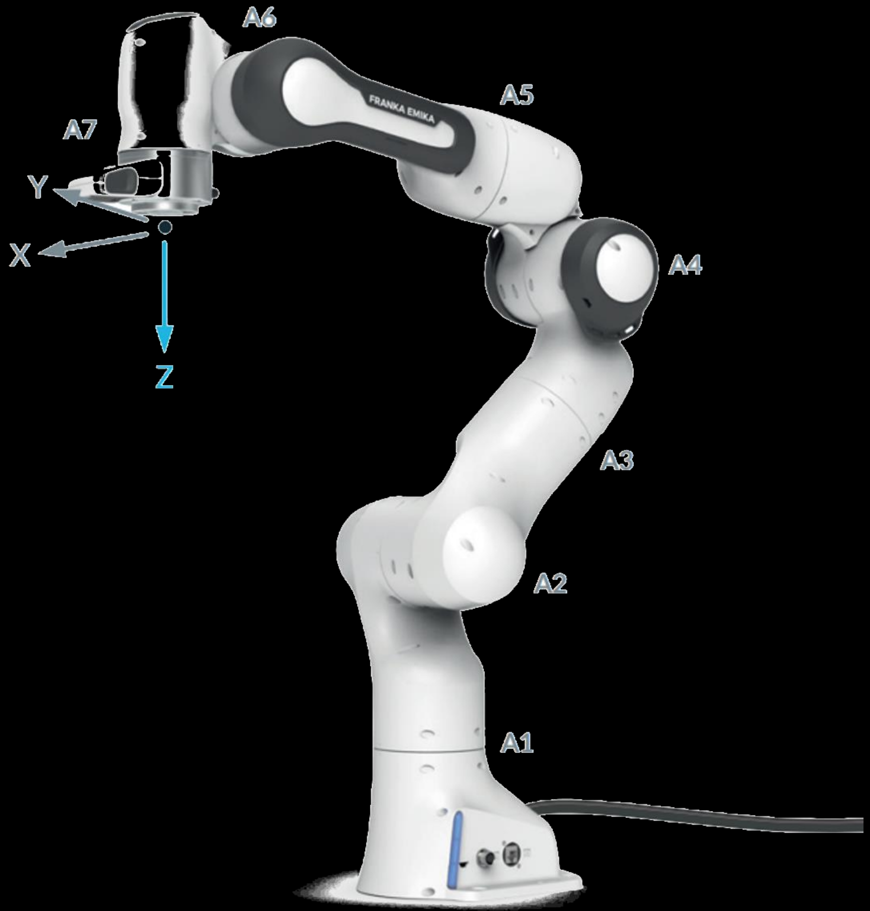
Franka Arm



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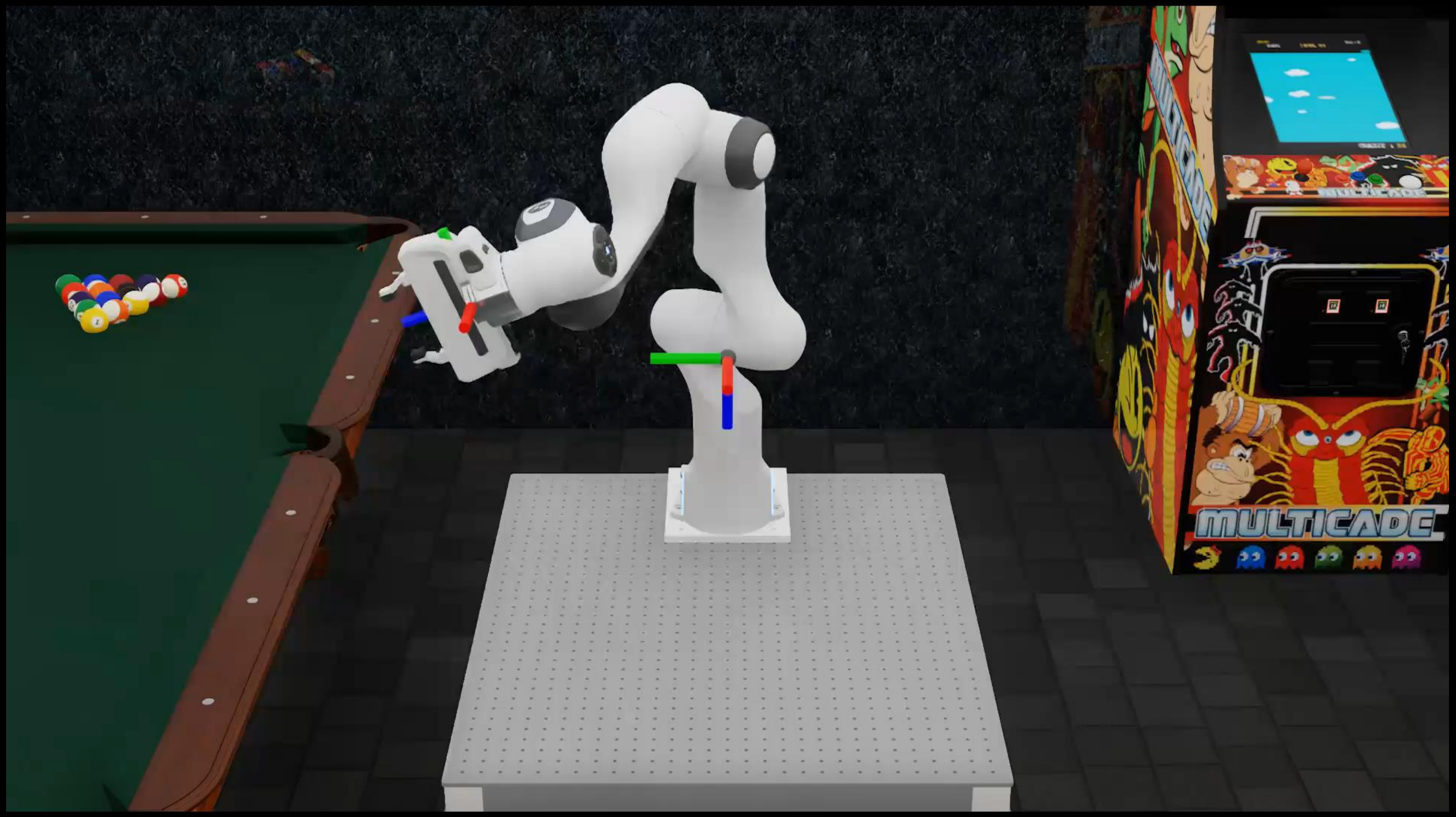
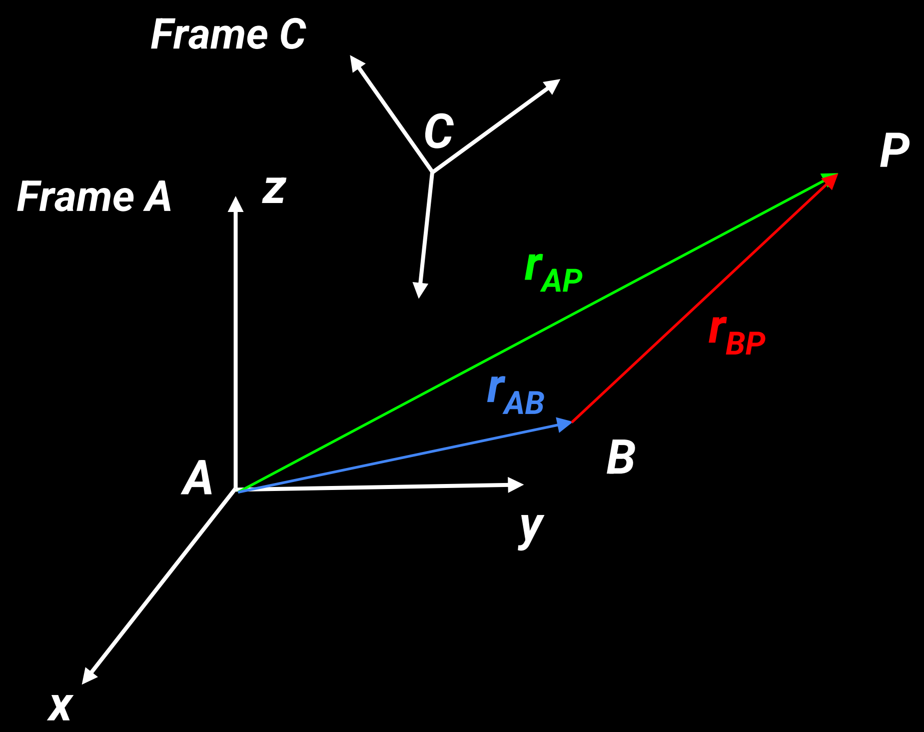
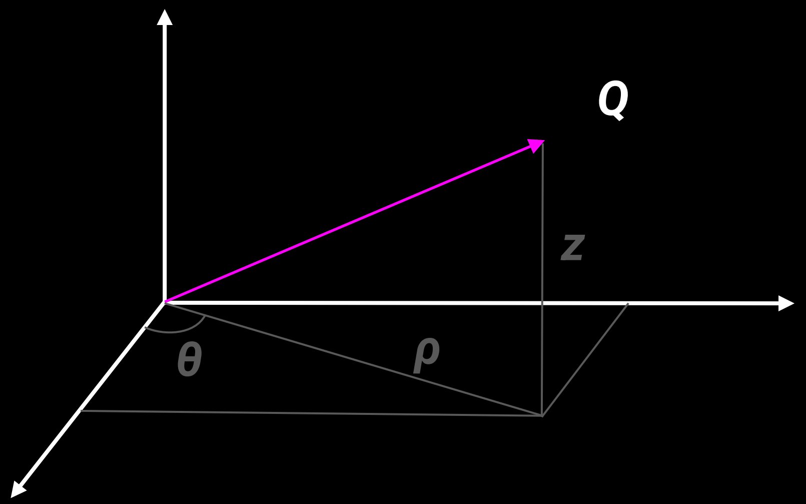


Videos from Orbit

Points, Lines, and Coordinates





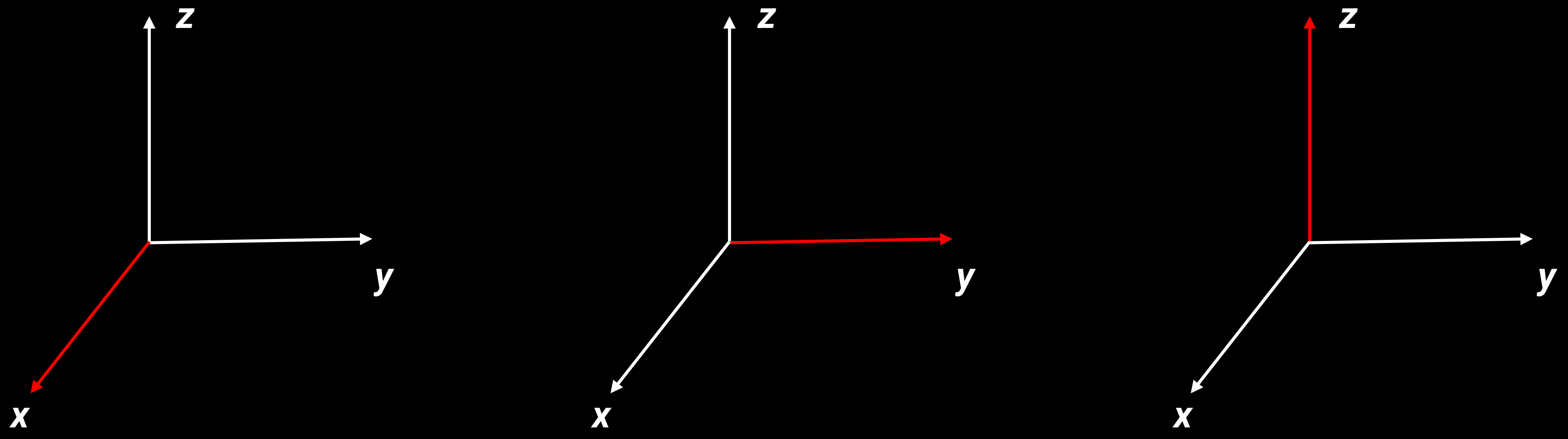
Point P in Cartesian Coordinates Frame A: $_{A}X_{P} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$

Point Q in Cylindrical Coordinate: $X_{Q} = \begin{pmatrix} \rho \\ \theta \\ z \end{pmatrix}$

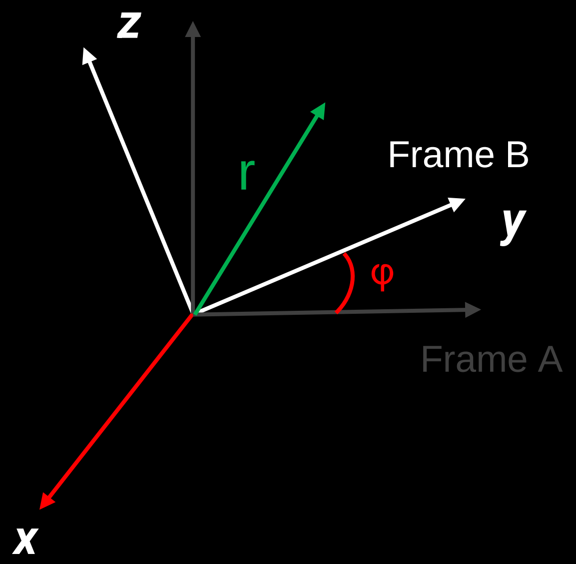
$$_{A}r_{AP} = {}_{A}r_{AB} + {}_{A}r_{BP}$$

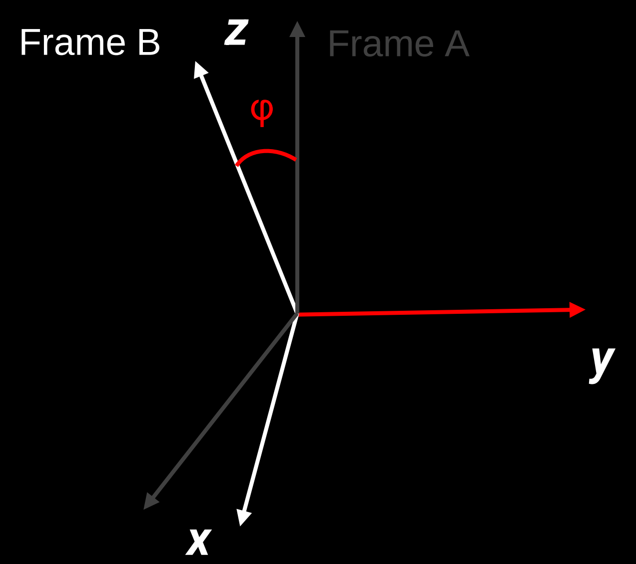
$$_{A}r_{AP} \neq {}_{A}r_{AB} + {}_{C}r_{BP}$$

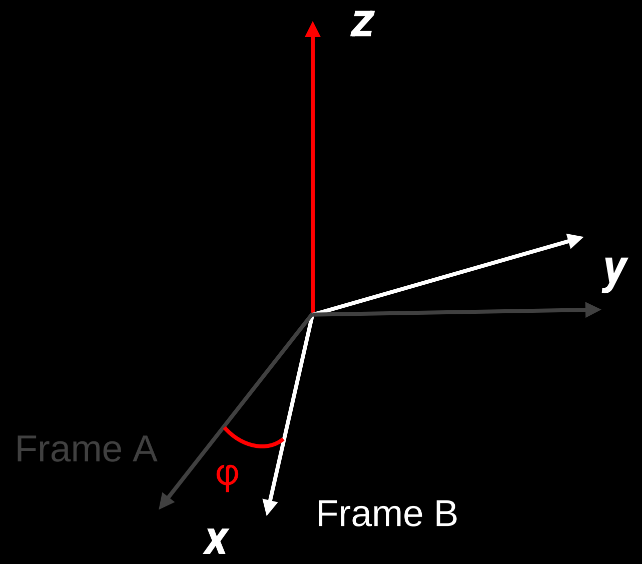
Rotation



Rotation







$$C_x(\varphi) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & cos\varphi & -sin\varphi \\ 0 & sin\varphi & cos\varphi \end{bmatrix}$$

$$C_y(\varphi) = \begin{bmatrix} cos\varphi & 0 & sin\varphi \\ 0 & 1 & 0 \\ -sin\varphi & 0 & cos\varphi \end{bmatrix}$$

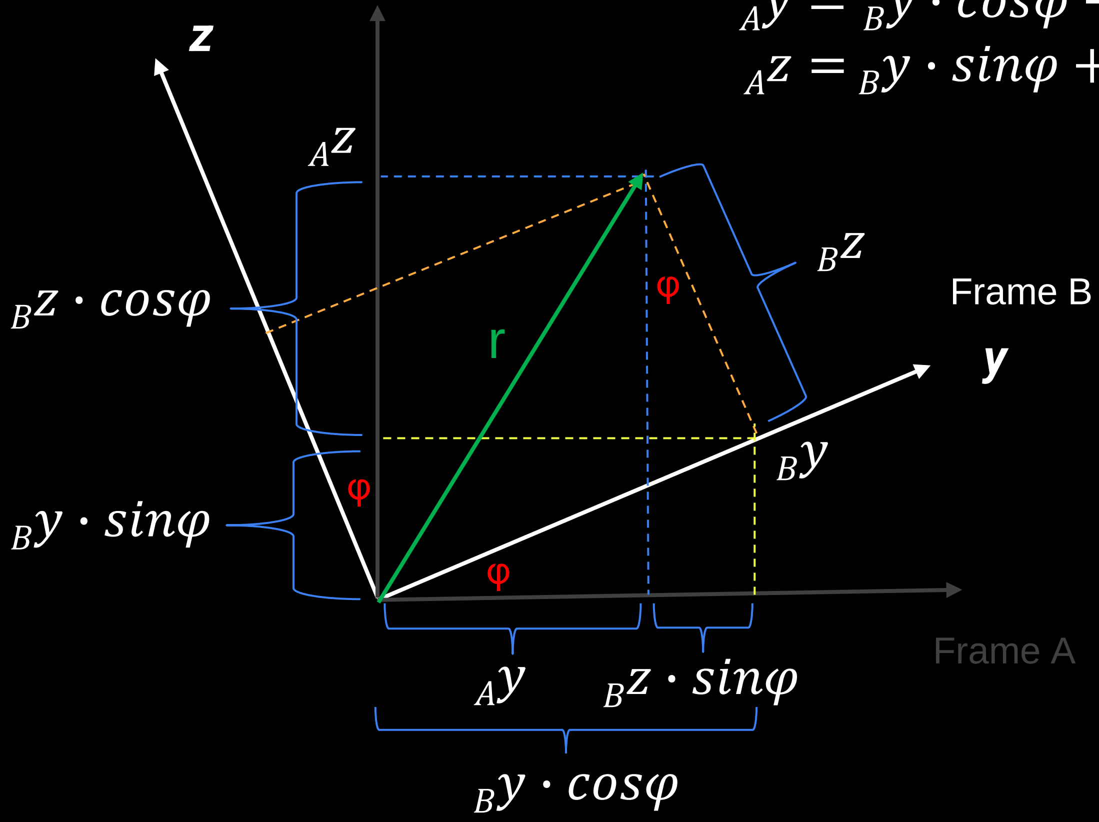
$$C_z(\varphi) = \begin{bmatrix} cos\varphi & -sin\varphi & 0 \\ sin\varphi & cos\varphi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

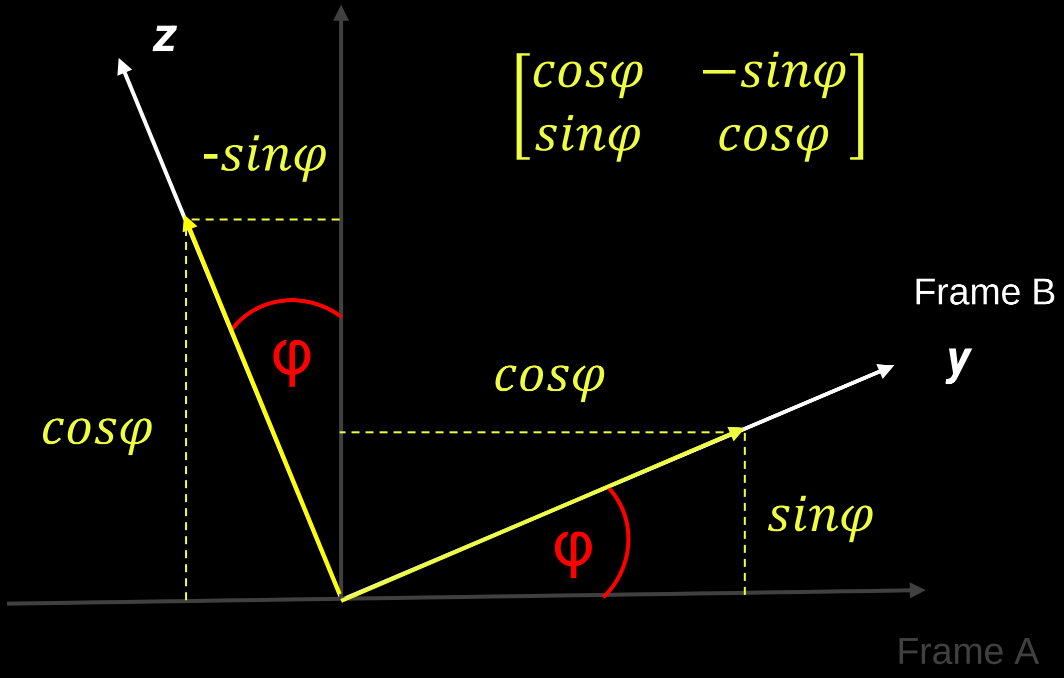
$$_A r = C_{AB} \cdot {_B r} \rightarrow \begin{pmatrix} _A x \\ _A y \\ _A z \end{pmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & cos\varphi & -sin\varphi \\ 0 & sin\varphi & cos\varphi \end{bmatrix} \begin{pmatrix} _B x \\ _B y \\ _B z \end{pmatrix} = \begin{pmatrix} _B x \\ _B y \cdot cos\varphi - {_B z} \cdot sin\varphi \\ _B y \cdot sin\varphi + {_B z} \cdot cos\varphi \end{pmatrix}$$

Rotation

$$_A r = C_{AB} \cdot {_B r} \rightarrow \begin{pmatrix} _A x \\ _A y \\ _A z \end{pmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & cos\varphi & -sin\varphi \\ 0 & sin\varphi & cos\varphi \end{bmatrix} \begin{pmatrix} _B x \\ _B y \\ _B z \end{pmatrix} = \begin{pmatrix} _B x \\ _B y \cdot cos\varphi - {_B z} \cdot sin\varphi \\ _B y \cdot sin\varphi + {_B z} \cdot cos\varphi \end{pmatrix}$$

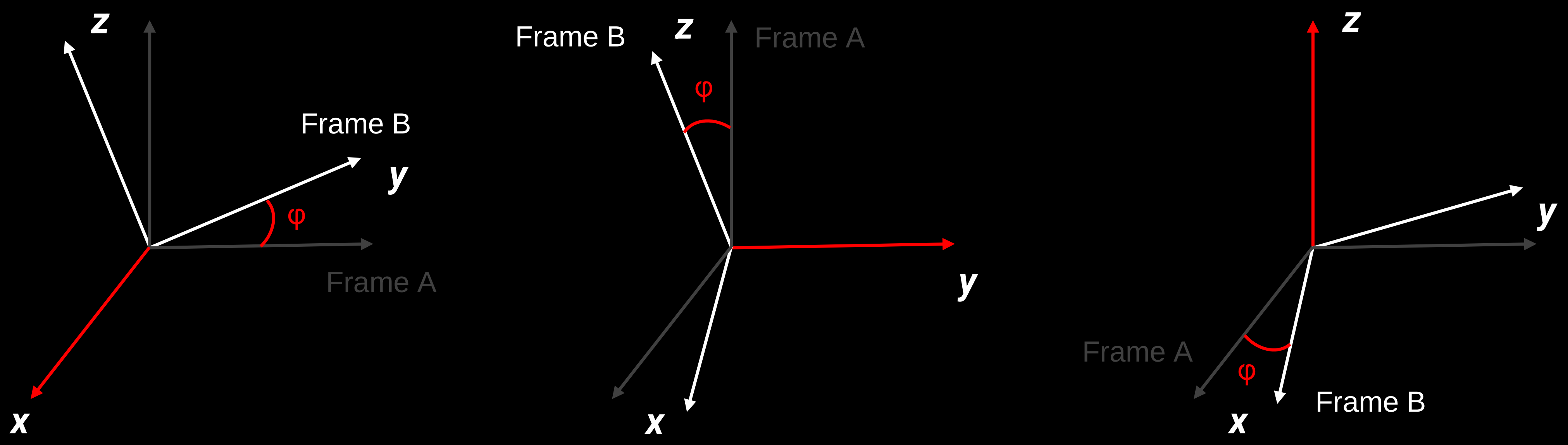
$$_A y = {_B y} \cdot cos\varphi - {_B z} \cdot sin\varphi$$
$$_A z = {_B y} \cdot sin\varphi + {_B z} \cdot cos\varphi$$





(only looking at y & z here)

Rotation



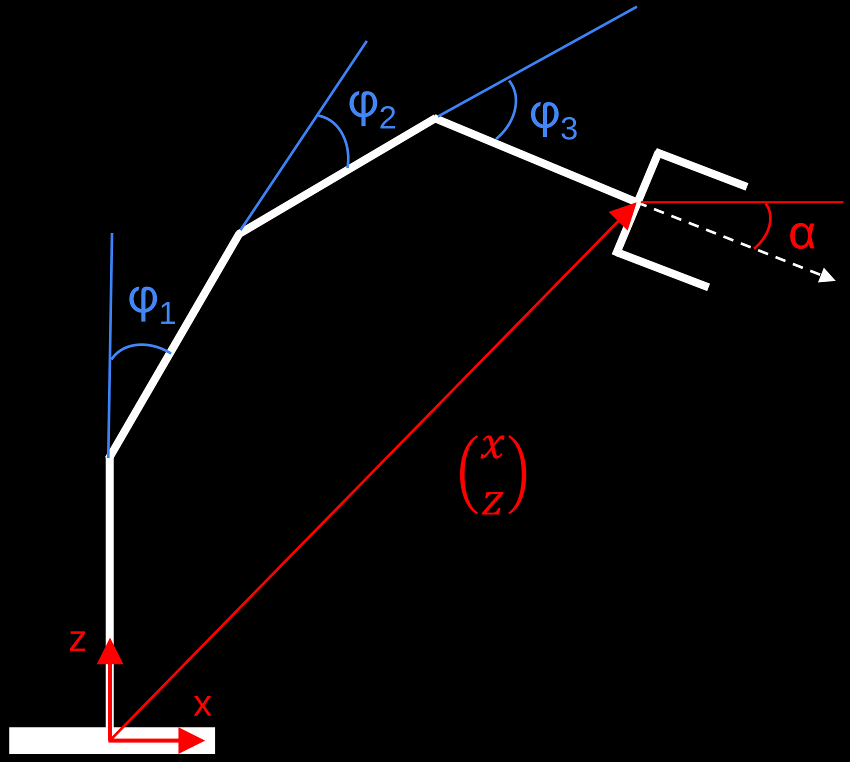
$$C_x(\varphi) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & cos\varphi & -sin\varphi \\ 0 & sin\varphi & cos\varphi \end{bmatrix} \quad C_y(\varphi) = \begin{bmatrix} cos\varphi & 0 & sin\varphi \\ 0 & 1 & 0 \\ -sin\varphi & 0 & cos\varphi \end{bmatrix} \quad C_z(\varphi) = \begin{bmatrix} cos\varphi & -sin\varphi & 0 \\ sin\varphi & cos\varphi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

If first rotate about z axis for z_1 angle, then about y axis for y angle, and lastly about z axis again for z_2 angle:

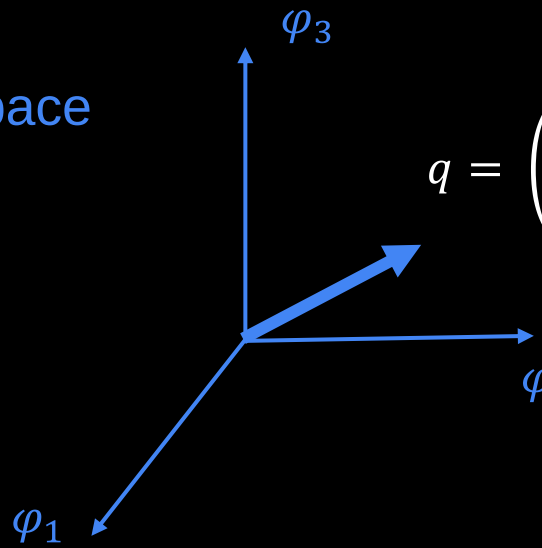
$$C_{AD} = C_{AB}(z_1)\, C_{BC}(y)\, C_{CD}(z_2) = \begin{bmatrix} cosz_1 & -sinz_1 & 0 \\ sinz_1 & cosz_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} cosy & 0 & siny \\ 0 & 1 & 0 \\ -siny & 0 & cosy \end{bmatrix} \begin{bmatrix} cosz_2 & -sinz_2 & 0 \\ sinz_2 & cosz_2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

↳ ${}_{A}r = C_{AD} \cdot {}_{D}r$

Joint Space and Task Space



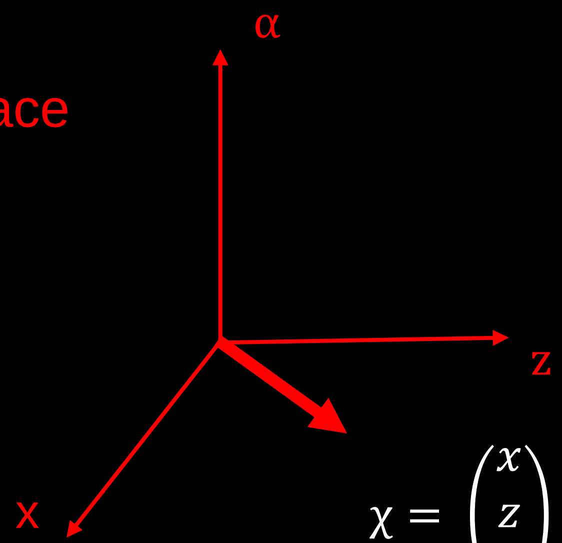
Joint Space



$$q = \begin{pmatrix} \varphi_1 \\ \varphi_2 \\ \varphi_3 \end{pmatrix}$$

Joint Coordinates

Task Space



$$\chi = \begin{pmatrix} x \\ z \\ \alpha \end{pmatrix}$$

Task Coordinates

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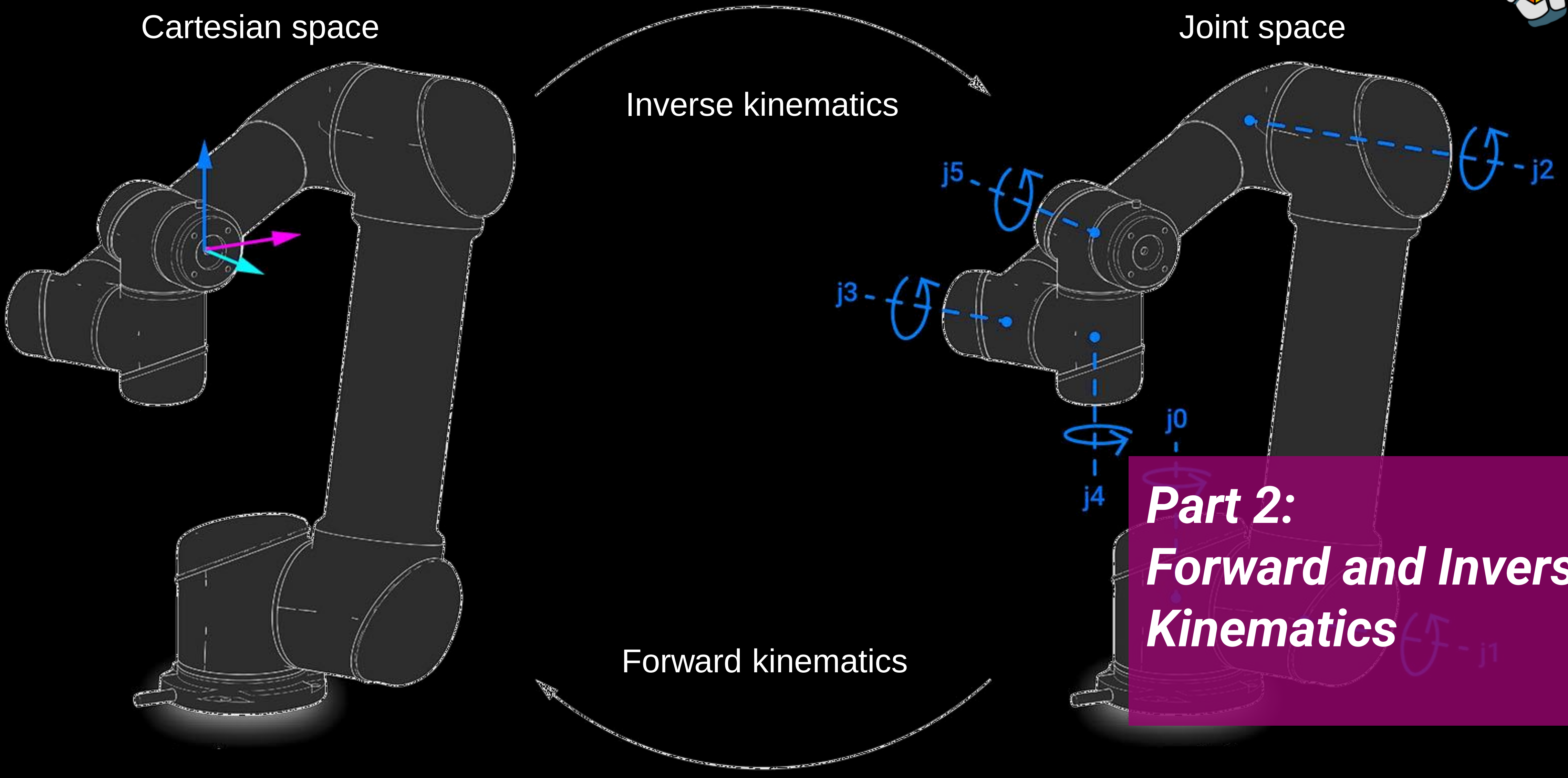

Part 2: Forward and Inverse Kinematics

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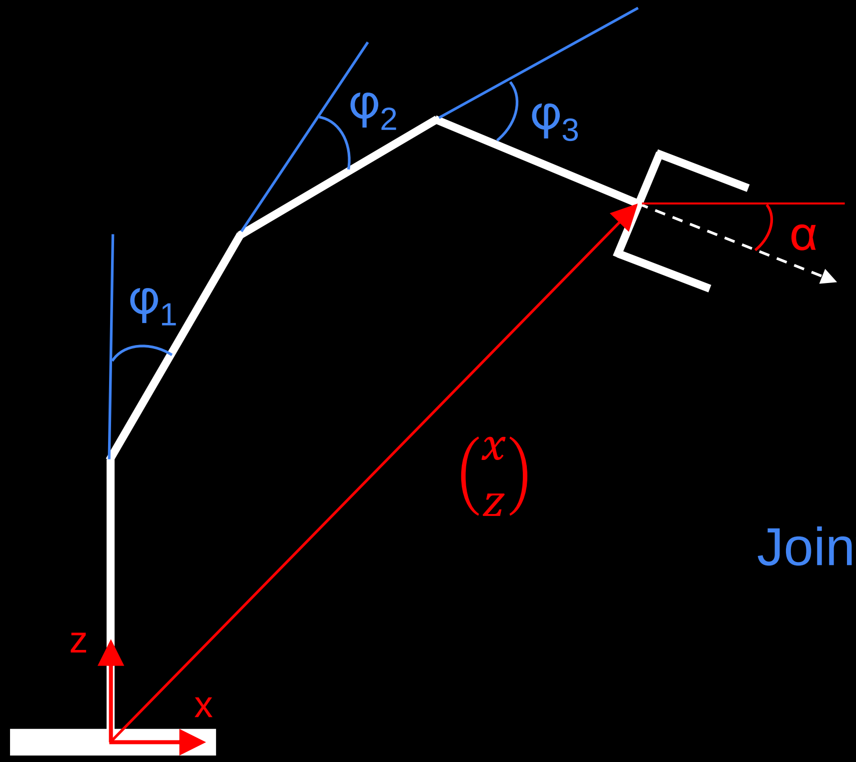
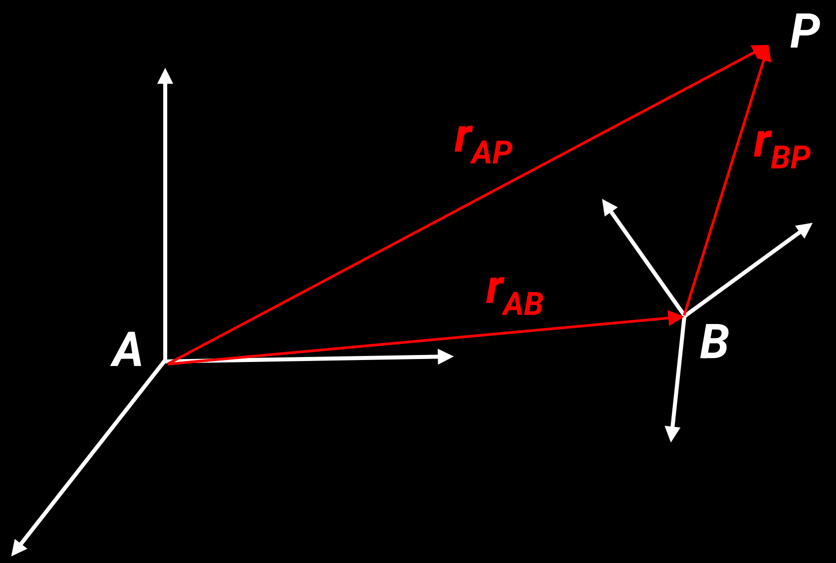
Forward and Inverse Kinematics



Joint Space —Forward Kinematics→ Task Space

Joint Space ←Inverse Kinematics— Task Space

Homogeneous Transformation Matrix



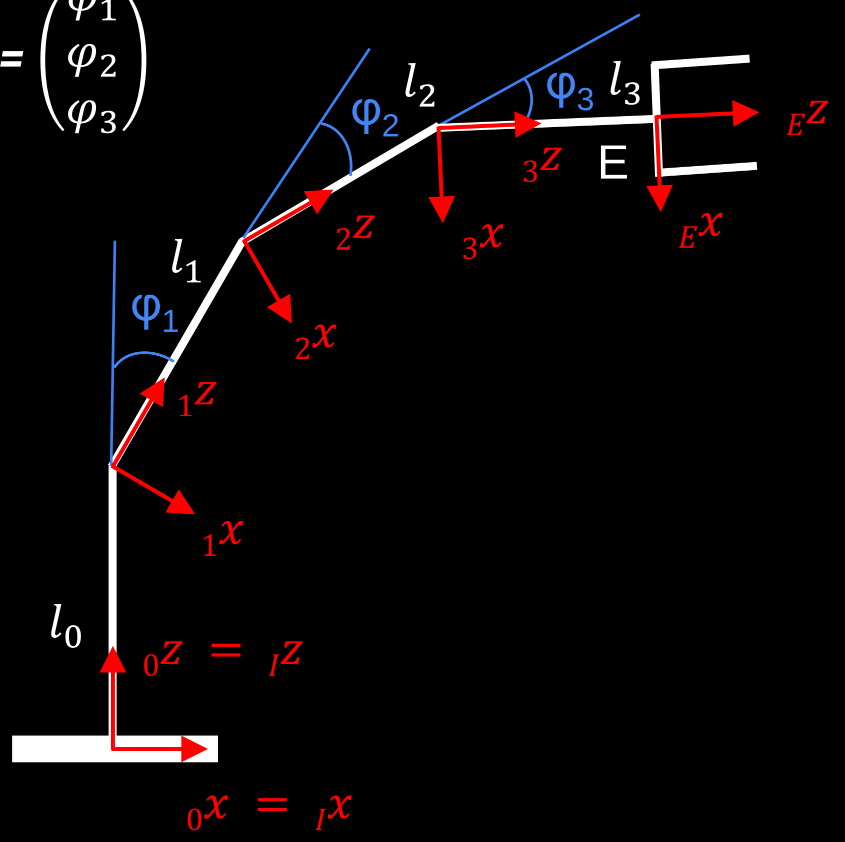
$$r_{AP} = r_{AB} + r_{BP}$$

$$_A r_{AP} = {}_A r_{AB} + {}_A r_{BP} = {}_A r_{AB} + C_{AB} \cdot {}_B r_{BP}$$

$$\begin{pmatrix} _A r_{AP} \\ 1 \end{pmatrix} = \underbrace{\begin{bmatrix} C_{AB} & _A r_{AB} \\ 0_{1x3} & 1 \end{bmatrix}}_{T_{AB}} \begin{pmatrix} _B r_{BP} \\ 1 \end{pmatrix}$$

Homogeneous Transformation Matrix

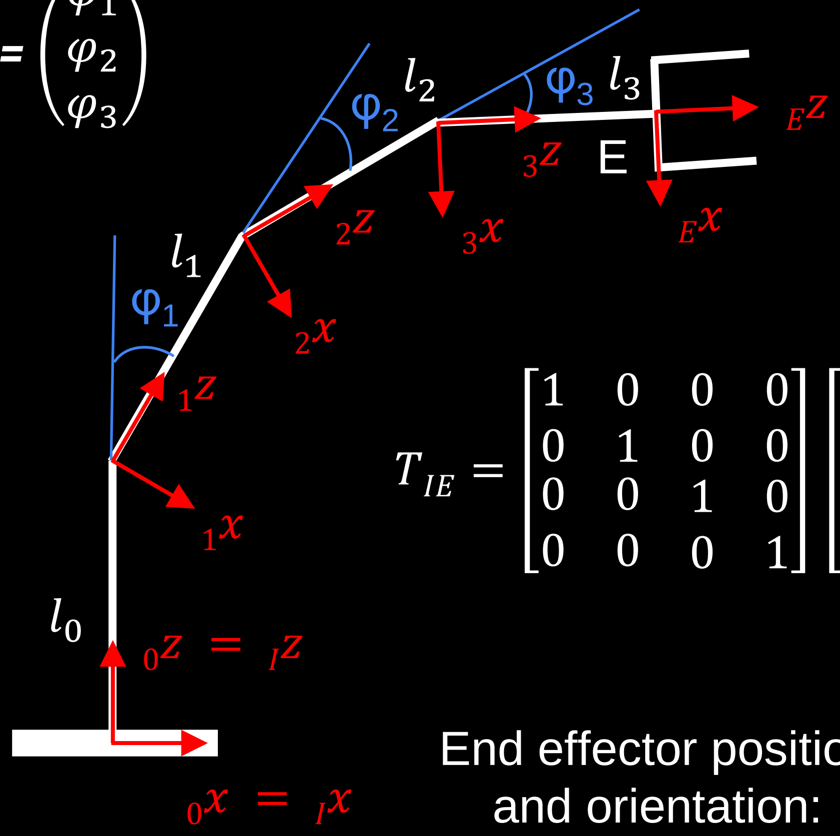
$\mathbf{q} = \begin{pmatrix} \varphi_1 \\ \varphi_2 \\ \varphi_3 \end{pmatrix}$



$$T_{IE} = TI_0 \cdot T_{01} \cdot T_{12} \cdot T_{23} \cdot T_{3E}$$

Homogeneous Transformation Matrix

$$\mathbf{q} = \begin{pmatrix} \varphi_1 \\ \varphi_2 \\ \varphi_3 \end{pmatrix}$$



$$T_{IE} = TI_0 \cdot T_{01} \cdot T_{12} \cdot T_{23} \cdot T_{3E}$$

$$T_{IE} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c_1 & 0 & s_1 & 0 \\ 0 & 1 & 0 & 0 \\ -s_1 & 0 & c_1 & l_0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c_2 & 0 & s_2 & 0 \\ 0 & 1 & 0 & 0 \\ -s_2 & 0 & c_2 & l_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c_3 & 0 & s_3 & 0 \\ 0 & 1 & 0 & 0 \\ -s_3 & 0 & c_3 & l_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & l_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

End effector position and orientation:

$$_I\mathbf{X}_E(\mathbf{q}) = \begin{pmatrix} l_1 \sin(\varphi_1) + l_2 \sin(\varphi_1 + \varphi_2) + l_3 \sin(\varphi_1 + \varphi_2 + \varphi_3) \\ l_0 + l_1 \cos(\varphi_1) + l_2 \cos(\varphi_1 + \varphi_2) + l_3 \cos(\varphi_1 + \varphi_2 + \varphi_3) \\ \varphi_1 + \varphi_2 + \varphi_3 \end{pmatrix}$$

Forward Differential Kinematics and Jacobian

$$\delta X_E \approx \frac{\delta X_E(q)}{\delta q}\delta q = J_{EA}(q)\delta q \qquad \text{with } J_{EA} = \frac{\delta X_E}{\delta q} = \begin{bmatrix} \frac{\delta X_1}{\delta q_1} & \cdots & \frac{\delta X_1}{\delta q_n} \\ \vdots & \ddots & \vdots \\ \frac{\delta X_m}{\delta q_1} & \cdots & \frac{\delta X_m}{\delta q_n} \end{bmatrix}$$

$$\dot{X}_E = J_{EA}(q)\dot{q} \qquad \text{with } J_{EA}(q) \in \mathbb{R}^{m\times n}$$

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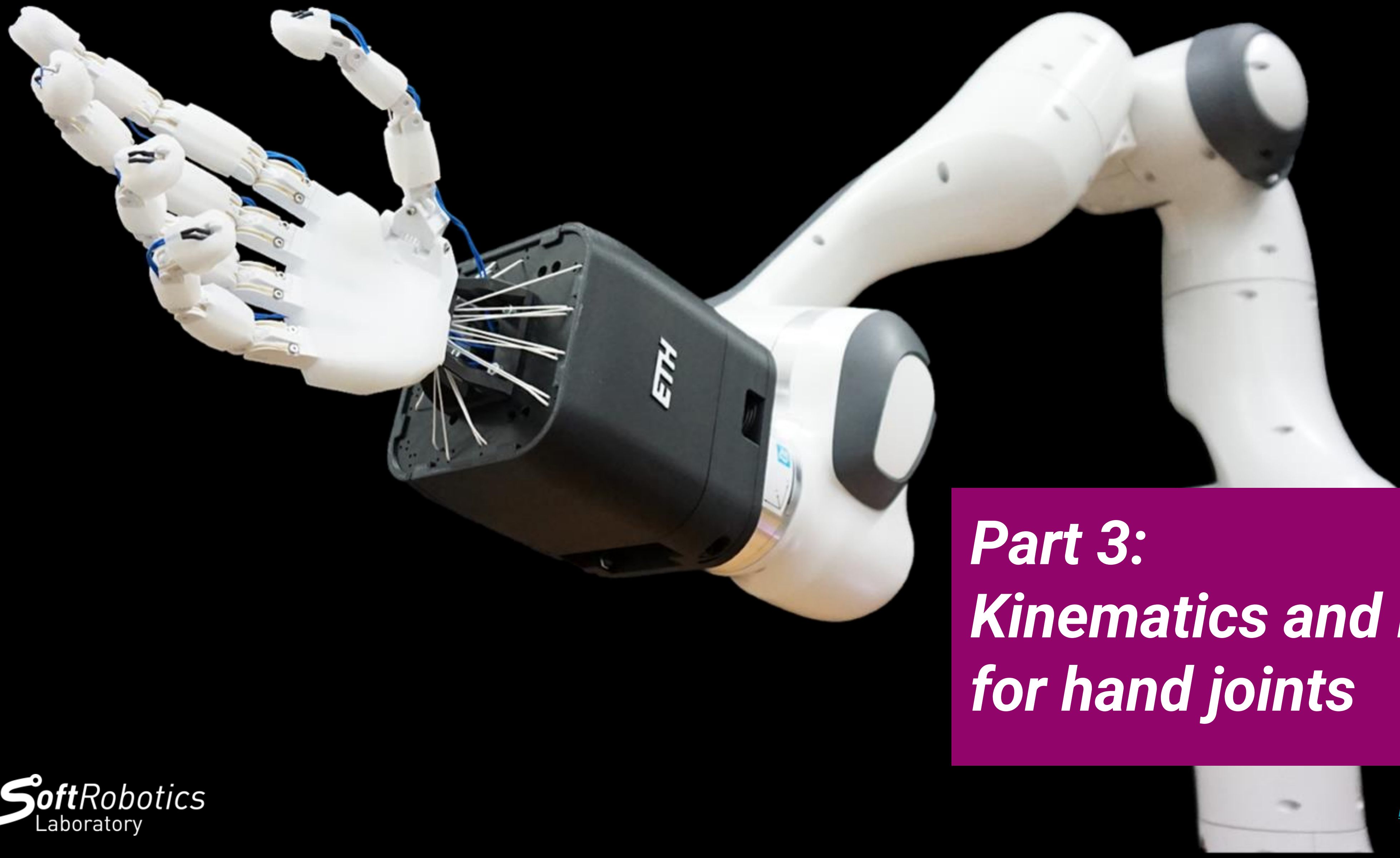

Part 3: Kinematics and Dynamics for hand joints

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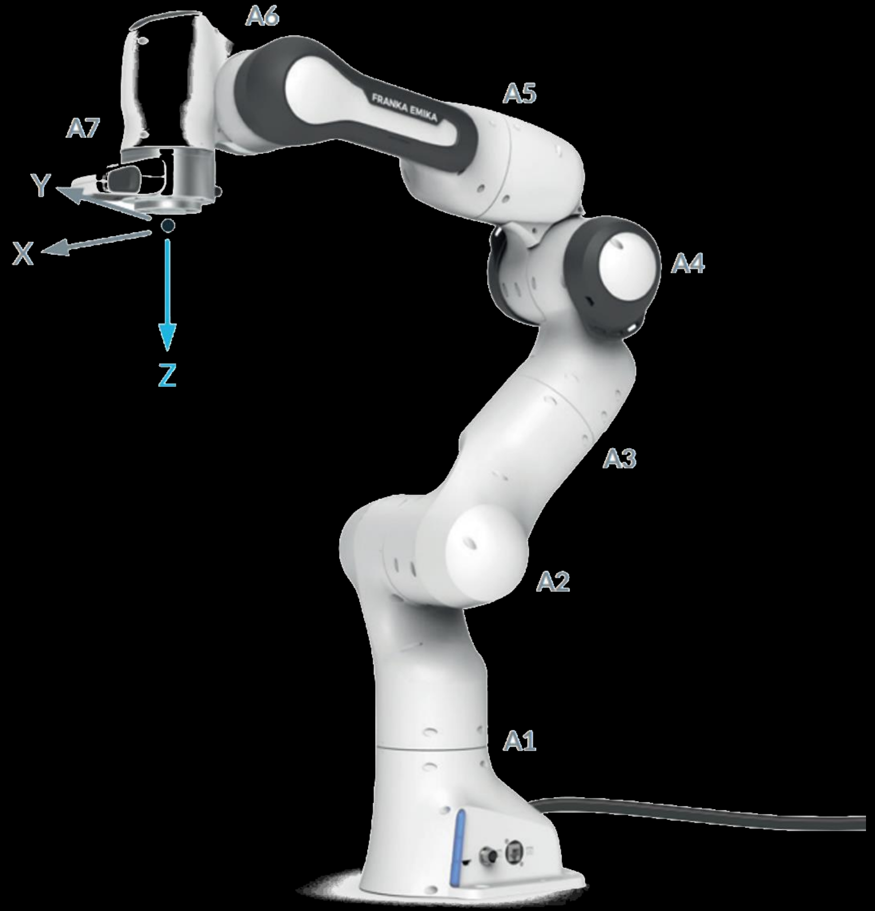
Difference Between Conventional Robots and Robotic Hands



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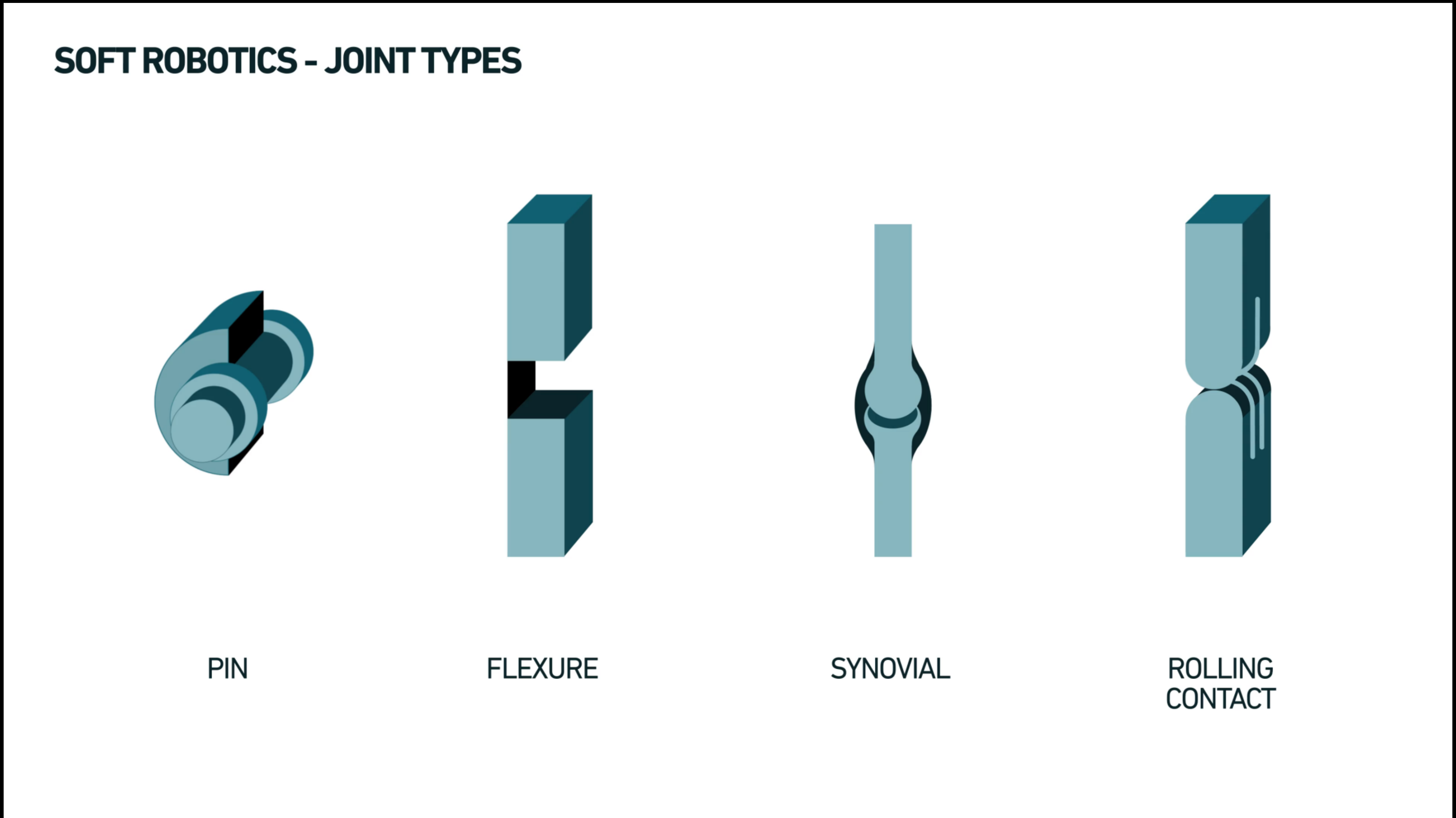
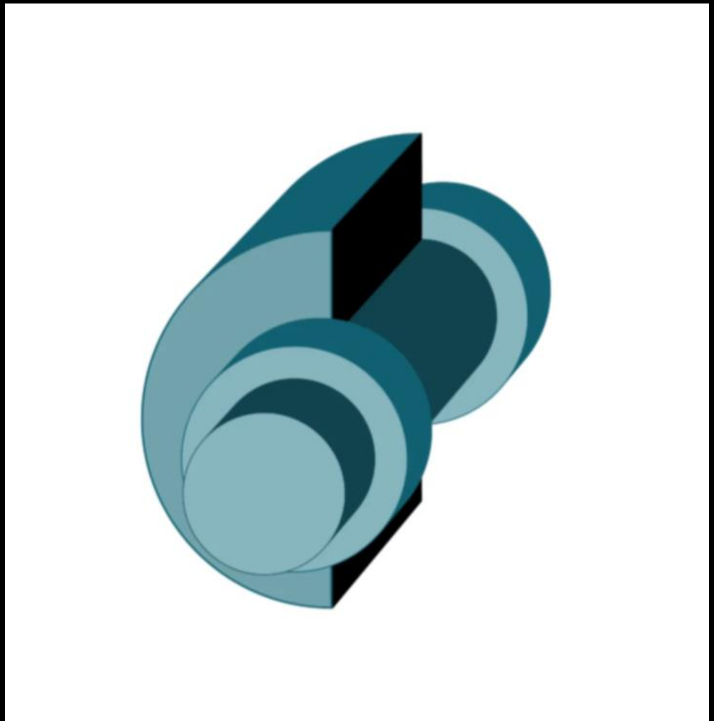
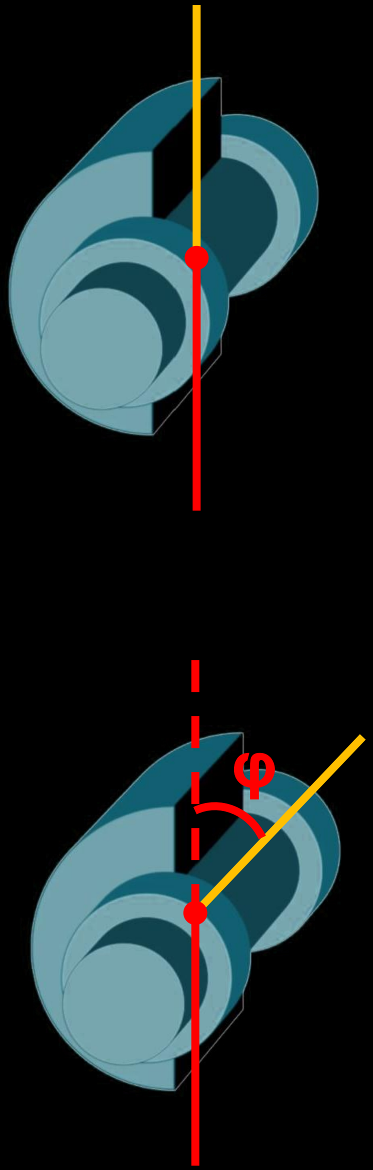
Recap: Different Types of Joints

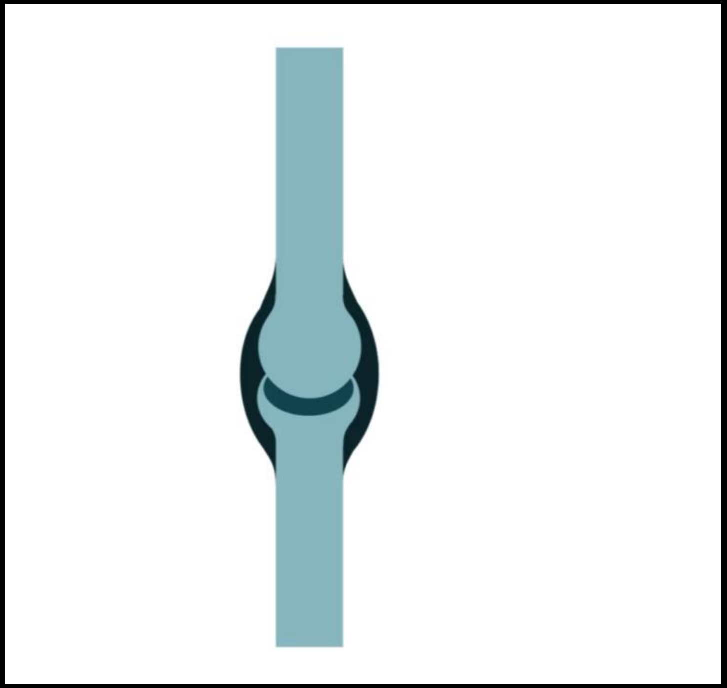


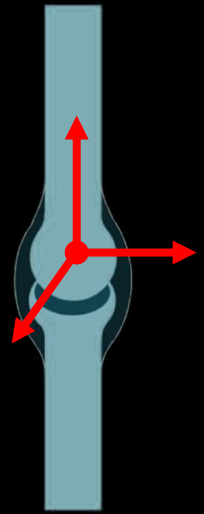
Pin Joint

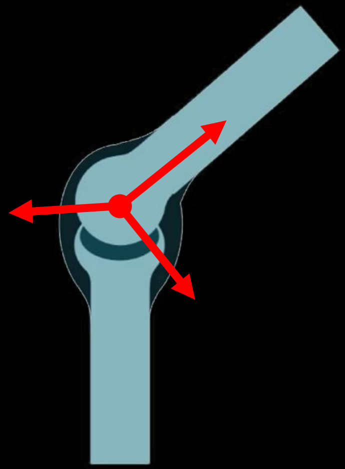




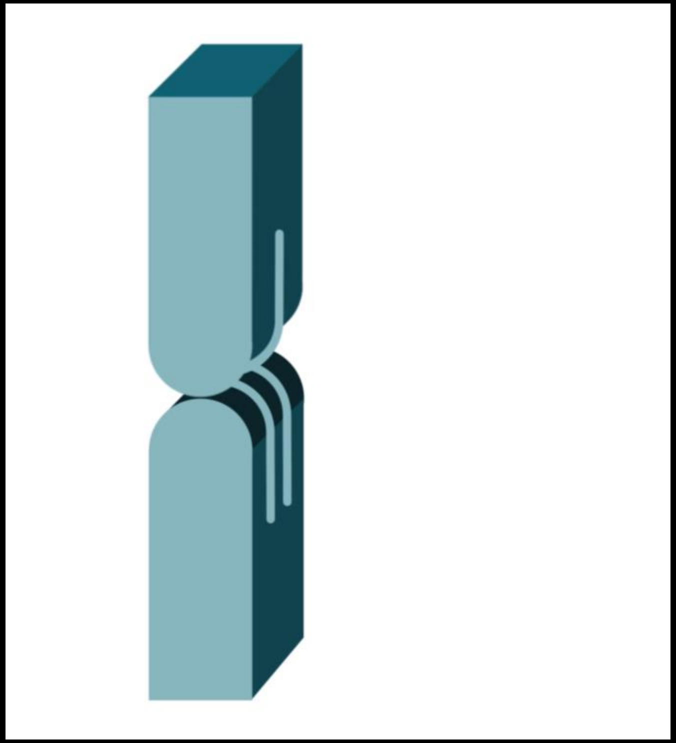
Synovial Joint

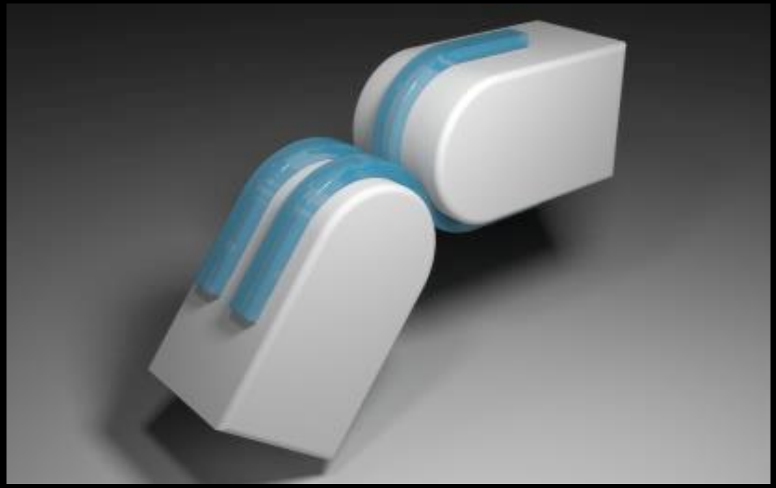


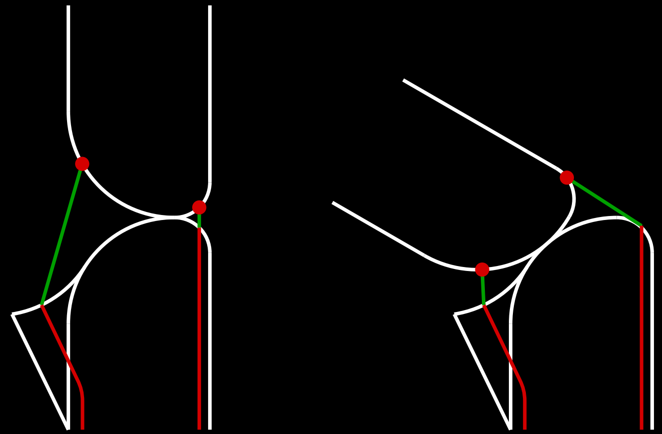




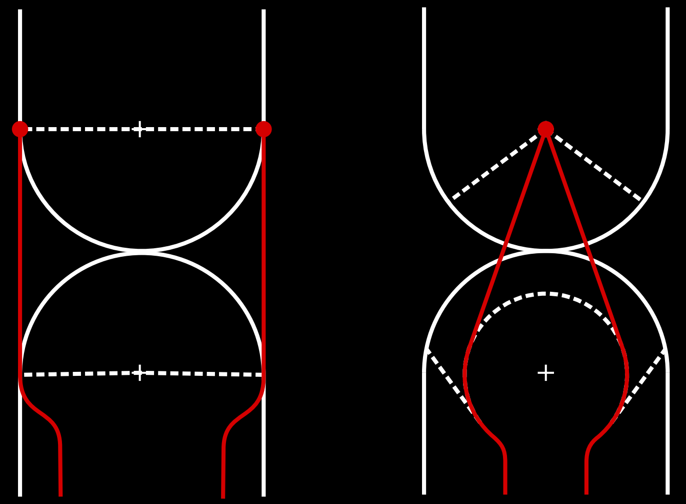
Rolling Contact Joint — Joints Used on Faive Hand

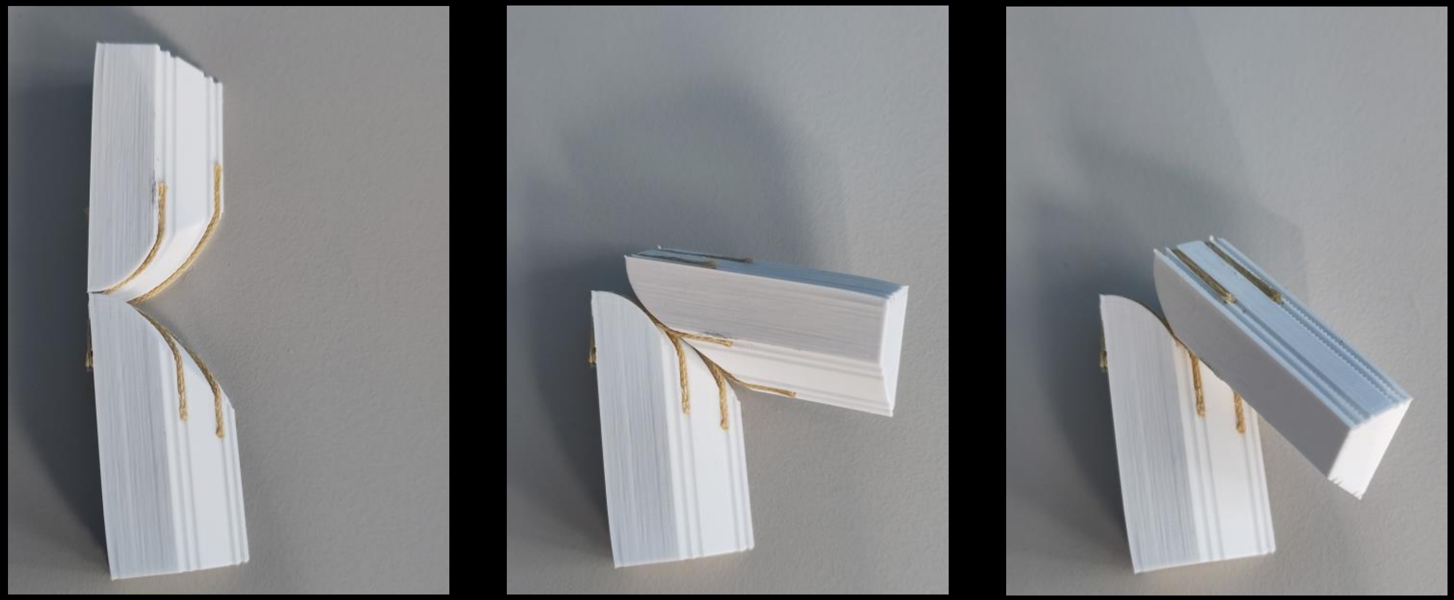




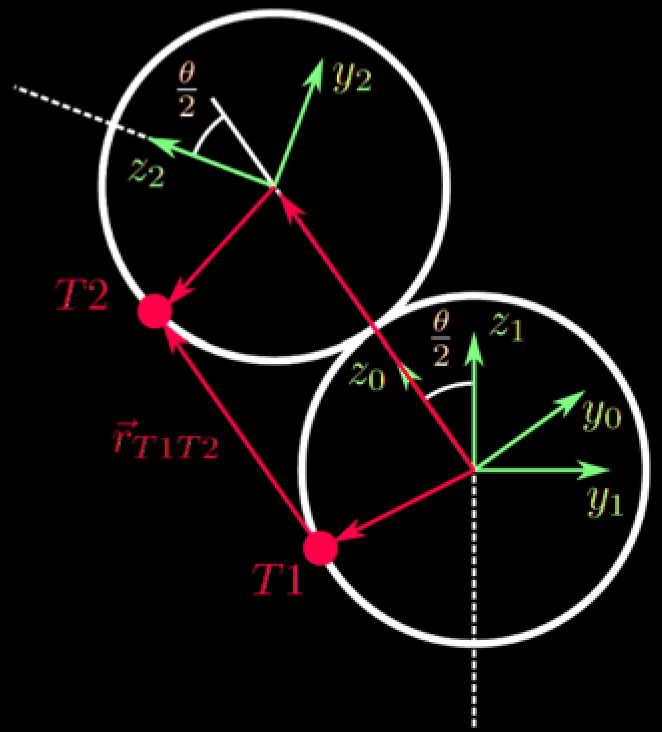


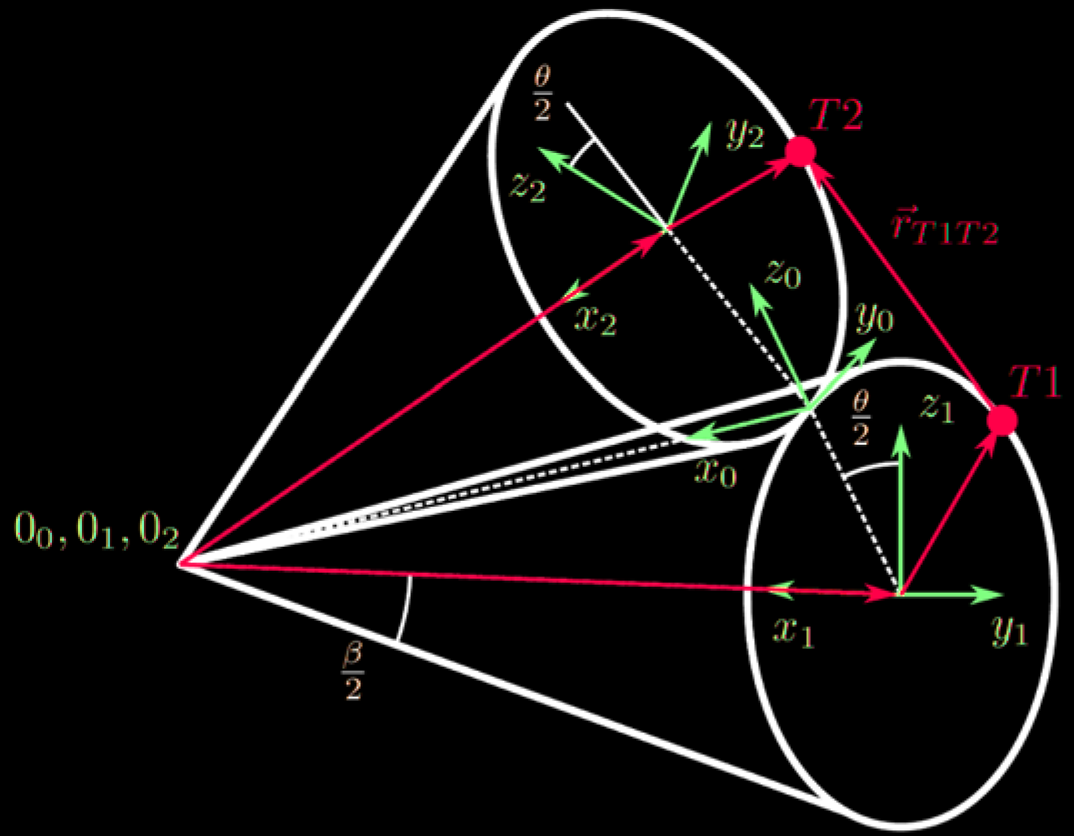
Kinematics for Rolling Contact Joint



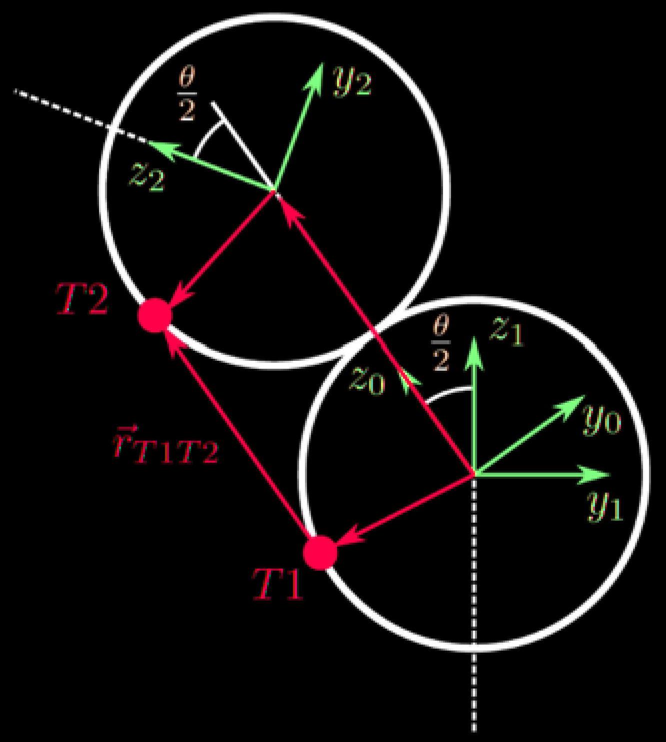


Kinematics for Rolling Contact Joint





Kinematics for Rolling Contact Joint



$$\vec{r}_{T1T2} = \begin{pmatrix} -X_1 \\ R_1 sin\alpha_1 \\ -R_1 cos\alpha_1 \end{pmatrix} + C_{10}\begin{pmatrix} 0 \\ 0 \\ 2R \end{pmatrix} + C_{12}\begin{pmatrix} X_2 \\ -R_2 sin\alpha_2 \\ R_2 cos\alpha_2 \end{pmatrix}$$

R: radius of the rolling cylinders

R_1, α_1 and X_1: cylinder coordinates of the point T1 in system 1

R_2, α_2 and X_2: cylinder coordinates of the point T2 in system 2

$$C_{10} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & cos\frac{\theta}{2} & -sin\frac{\theta}{2} \\ 0 & sin\frac{\theta}{2} & cos\frac{\theta}{2} \end{bmatrix}, \qquad C_{12} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & cos\theta & -sin\theta \\ 0 & sin\theta & cos\theta \end{bmatrix}$$

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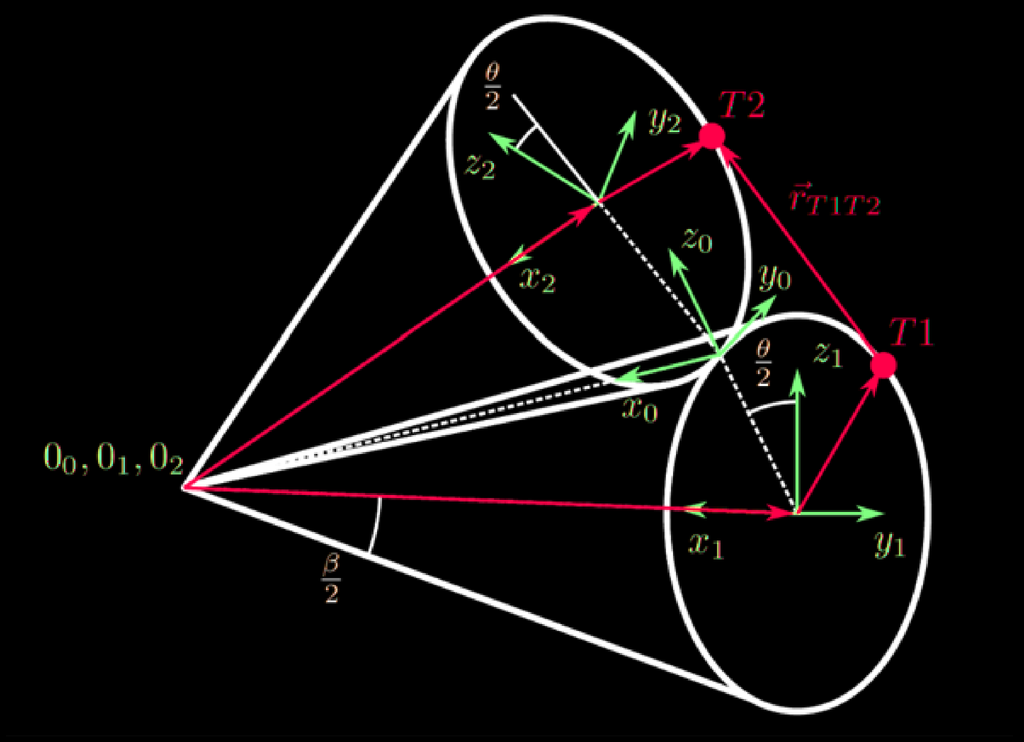
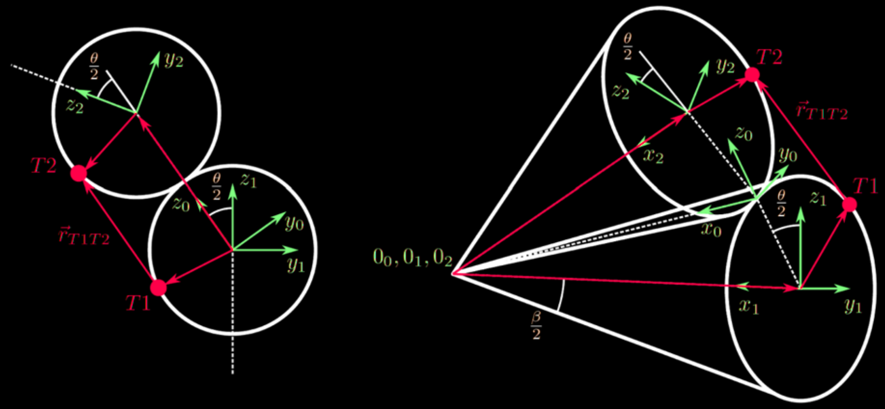
Kinematics for Rolling Contact Joint



$$\vec{r}_{T1T2} = \begin{pmatrix} -X_1 \\ R_1 sin\alpha_1 \\ -R_1 \cos\alpha_1 \end{pmatrix} + C_{12} \begin{pmatrix} X_2 \\ -R_2 sin\alpha_2 \\ R_2 cos\alpha_2 \end{pmatrix}$$

$$C_{12} = \begin{bmatrix} cos\beta & sin\frac{\theta}{2}sin\beta & cos\frac{\theta}{2}sin\beta \\ sin\frac{\theta}{2}sin\beta & cos\left(\frac{\theta}{2}\right)^2 - \sin\left(\frac{\theta}{2}\right)^2 cos\beta & -cos\frac{\theta}{2}sin\frac{\theta}{2}(1+cos\beta) \\ -cos\frac{\theta}{2}sin\beta & cos\frac{\theta}{2}sin\frac{\theta}{2}(1+cos\beta) & cos\left(\frac{\theta}{2}\right)^2 cos\beta - \sin\left(\frac{\theta}{2}\right)^2 \end{bmatrix}$$

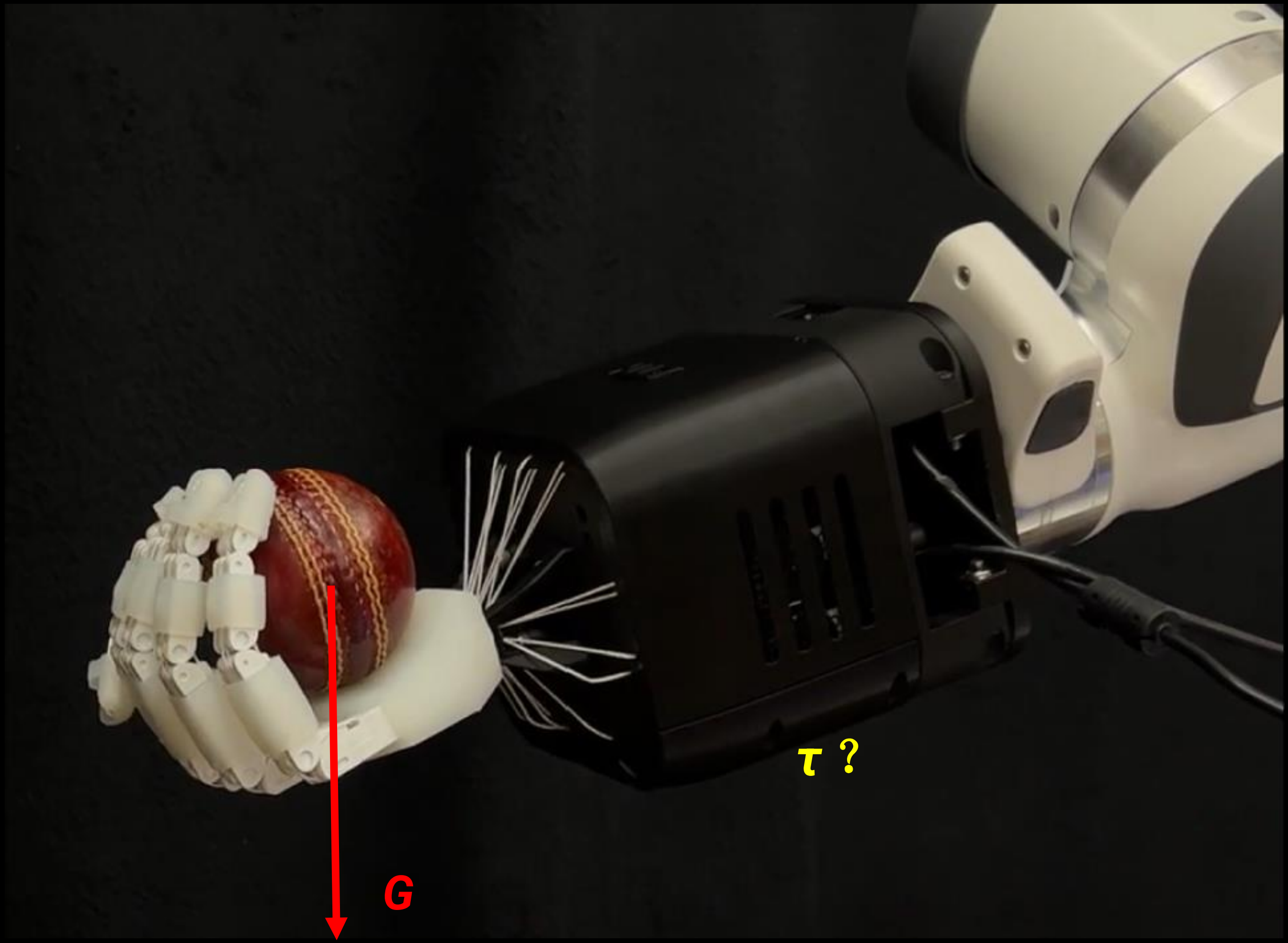
Kinematics for Rolling Contact Joint



$$\vec{r}_{T1T2} = \begin{pmatrix} -X_1 \\ R_1 sin\alpha_1 \\ -R_1 cos\alpha_1 \end{pmatrix} + C_{10}\begin{pmatrix} 0 \\ 0 \\ 2R \end{pmatrix} + C_{12}\begin{pmatrix} X_2 \\ -R_2 sin\alpha_2 \\ R_2 cos\alpha_2 \end{pmatrix}$$

$$C_{10} = \begin{bmatrix} cos\frac{\beta}{2} & 0 & sin\frac{\beta}{2} \\ sin\frac{\beta}{2}sin\frac{\theta}{2} & cos\frac{\theta}{2} & -cos\frac{\beta}{2}sin\frac{\theta}{2} \\ -sin\frac{\beta}{2}cos\frac{\theta}{2} & sin\frac{\theta}{2} & cos\frac{\beta}{2}cos\frac{\theta}{2} \end{bmatrix}$$

Dynamics



Dynamics

Tendon Lengths

$$p = g(l) = g\big(f(q)\big) = F(q)$$

Motor Positions

Joint Angles

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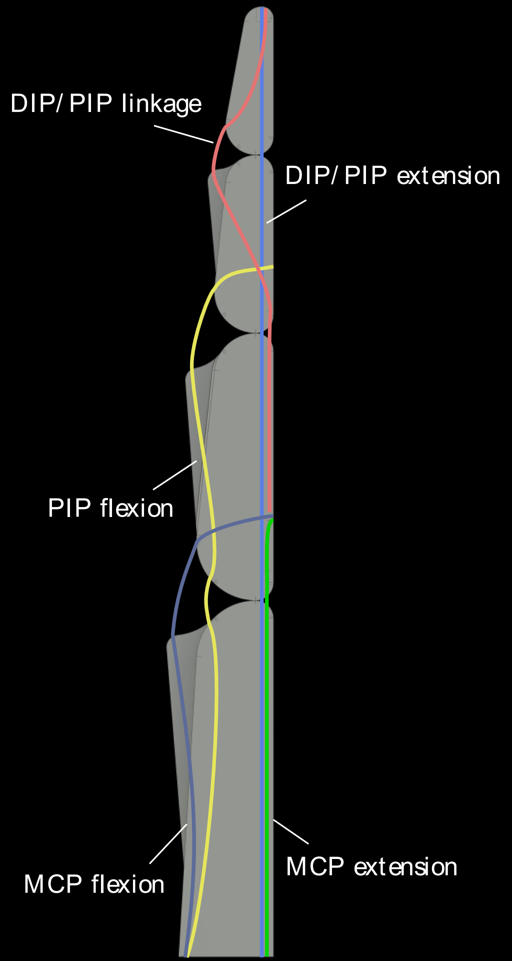
Dynamics

$$J_m = \begin{bmatrix} \frac{\partial p_1}{\partial q_1} & \frac{\partial p_1}{\partial q_2} \\ \frac{\partial p_2}{\partial q_1} & \frac{\partial p_2}{\partial q_2} \end{bmatrix}$$



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Dynamics

Velocity of the finger joints

$$\dot{p} = J_m \cdot \dot{q}$$

Velocity of the motors

$$\tau^T \cdot \dot{q} = T^T \cdot \dot{p}$$

Conservation of Power

$$\tau^T \cdot \dot{q} = T^T \cdot J_m \cdot \dot{q} \quad \rightarrow \quad \tau = {J_m}^T \cdot T$$

Dynamics

Previous slide: $\tau = {J_m}^T \cdot T$

$$\dot{X}_{fingertip} = Jfing_{ertip} \cdot \dot{q}$$

$$\tau^T \cdot \dot{q} = {F_{fingertip}}^T \cdot \dot{X}_{fingertip}$$

$$\tau^T \cdot \dot{q} = {F_{fingertip}}^T \cdot J_{fingertip} \cdot \dot{q}$$

$$\tau = {J_{fingertip}}^T \cdot F_{fingertip}$$

Dynamics

$$\tau = {J_m}^T \cdot T$$

$$\tau = {J_{fingertip}}^T \cdot F_{fingertip}$$

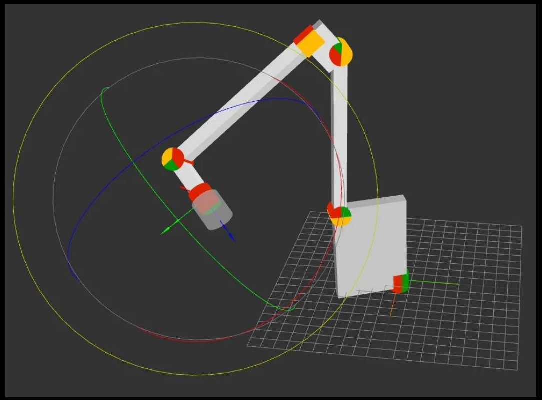
$$T = \left({J_m}^T\right)^{-1} \cdot {J_{fingertip}}^T \cdot F_{fingertip}$$

Summary

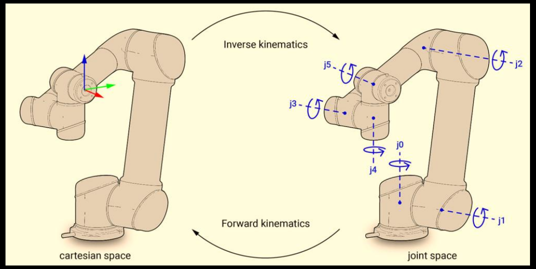
- *Intro to Robot Kinematics and Dynamics*
  - *Representing points and lines in different coordinates and frames*
  - *Rotational matrix*
  - *Joint space and task space*

- *Forward and Inverse Kinematics*
  - *Homogeneous transformation matrix*
  - *Forward differential kinematics and Jacobian*
  - *Inverse kinematics*

- *Kinematics and Dynamics for hand joints*
  - *Hand Joints*
  - *Kinematics for rolling joints*
  - *Dynamics for rolling joints*



Marginally Clever Robots



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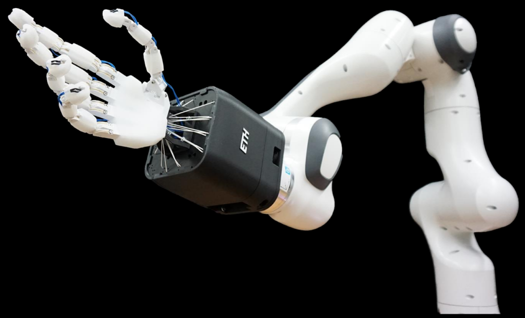

Faive Robotics

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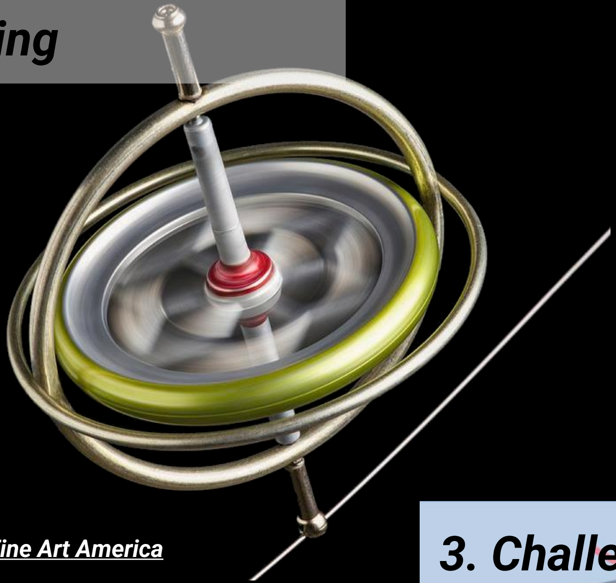
Next Tutorial? Implementing Control Strategies for Manipulation!

1. Sensing

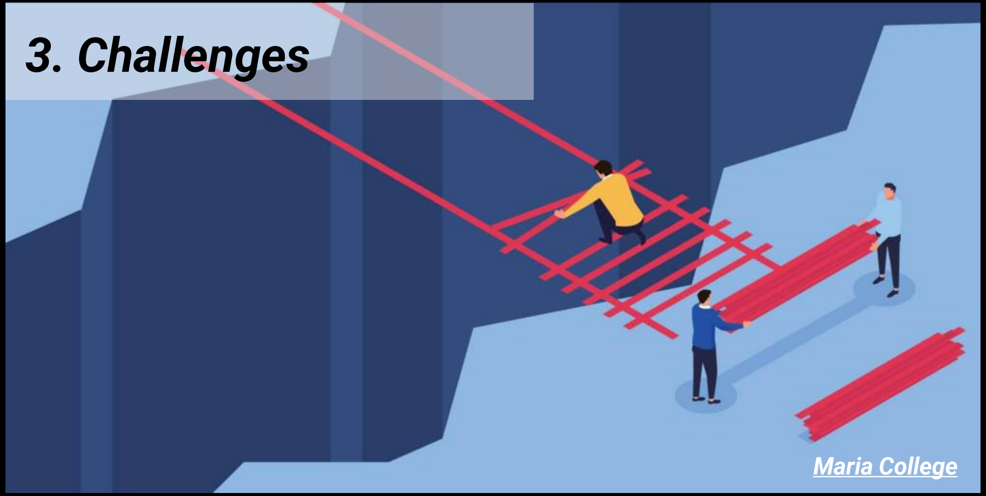


Fine Art America

2. Control

Wikimedia

3. Challenges



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