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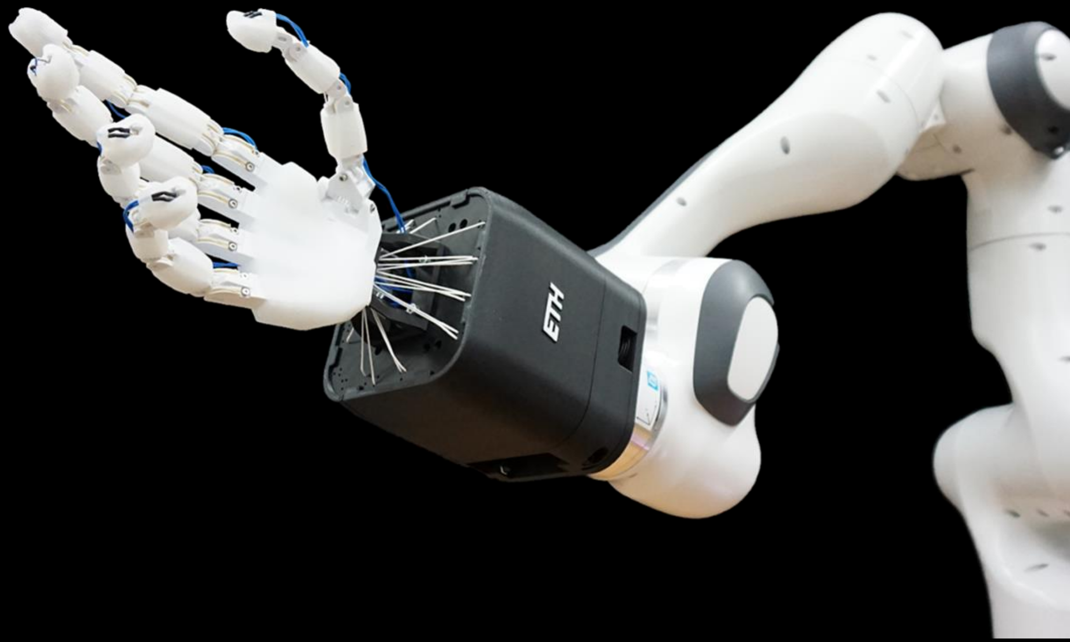
Kinematics, Dynamics and Control of Robotic Hands

Robert Katzschmann

Assistant Professor of Robotics, Soft Robotics Lab



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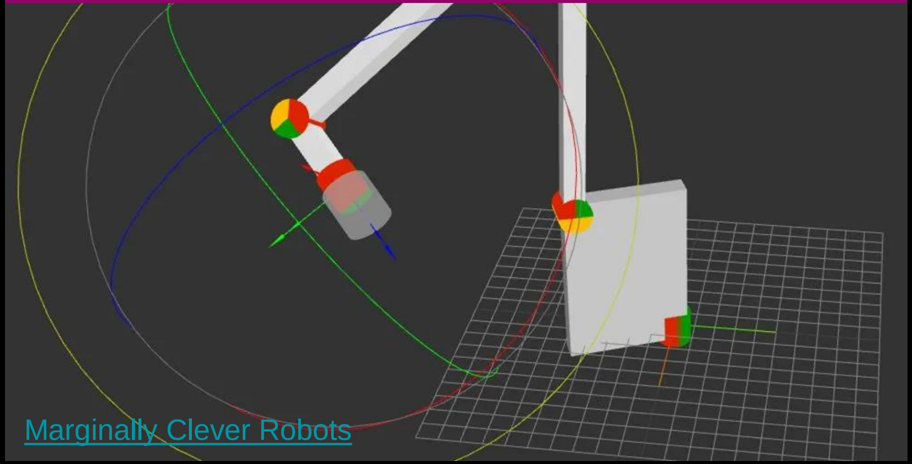
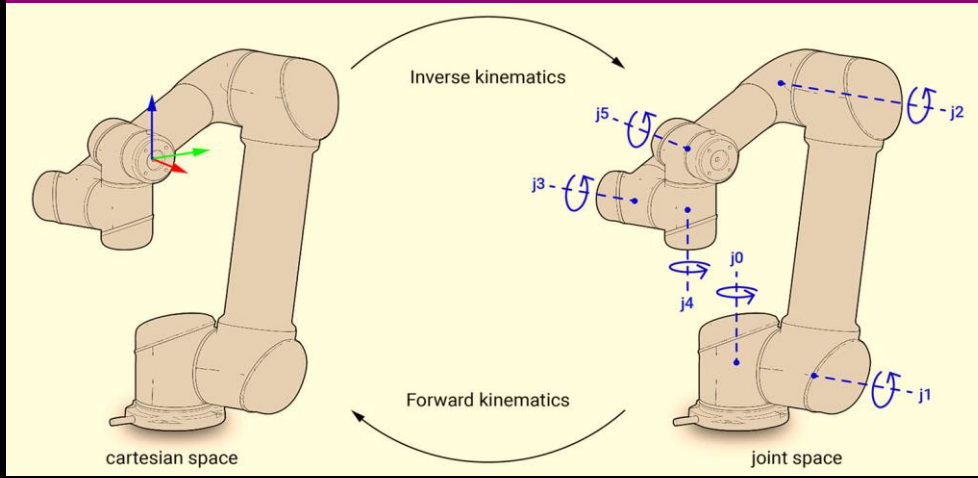
Focus Topics for Today

1. Intro to Robot Kinematics and Dynamics



Marginally Clever Robots

2. Forward and Inverse Kinematics



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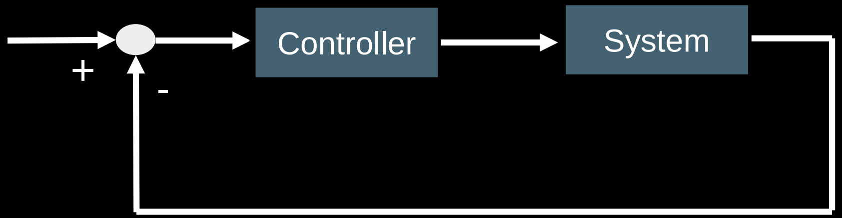
3. Kinematics and Dynamics for hand joints

Mimic Robotics

4. Control



5. Challenges

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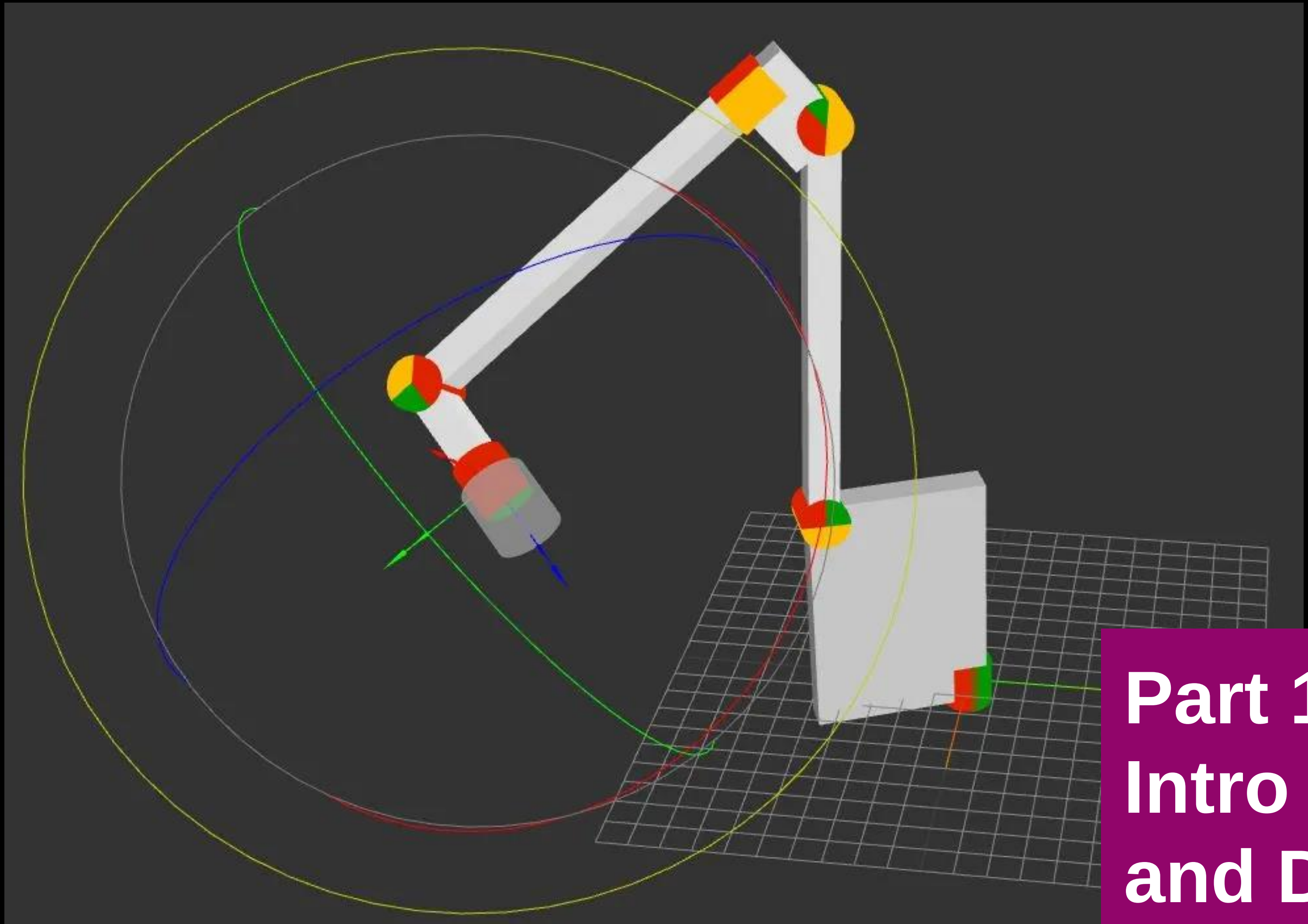
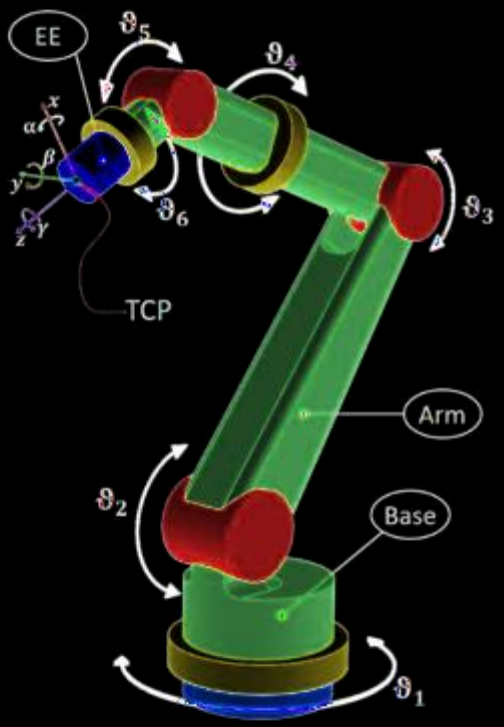


Marginally Clever Robots

Part 1: Intro to Robot Kinematics and Dynamics

Robot Kinematics and Dynamics



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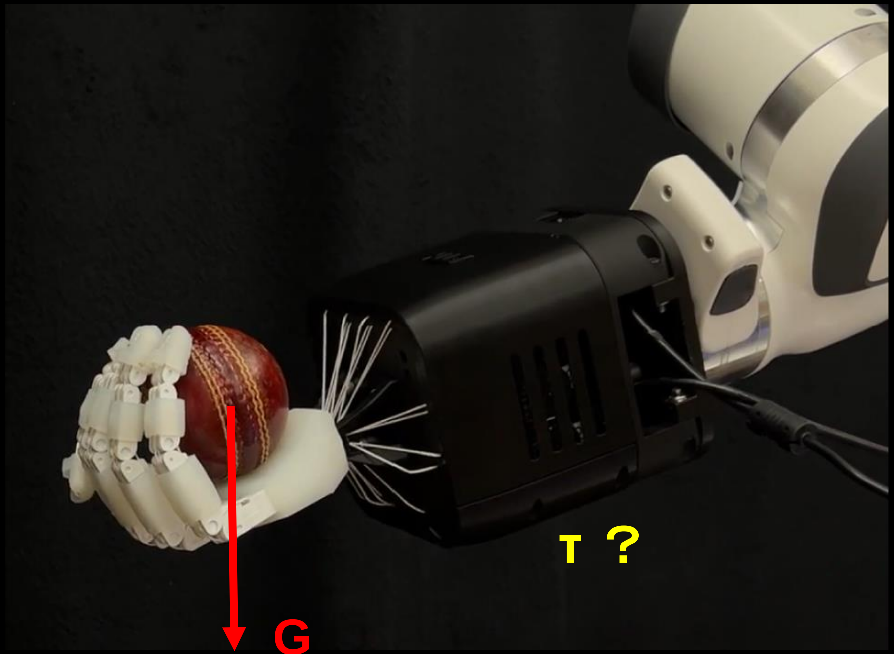
Kinematics



Toshimitsu et al. (2023) https://srl-ethz.github.io/get-ball-rolling/

Dynamics

Simulation
reaction to certain actuator commands

Control
invert of simulation, want to get somewhere, what to command?

Design
how are the loads distributed?

Optimization
what dimension should I have?

Actuation
torque, speed, powder etc.

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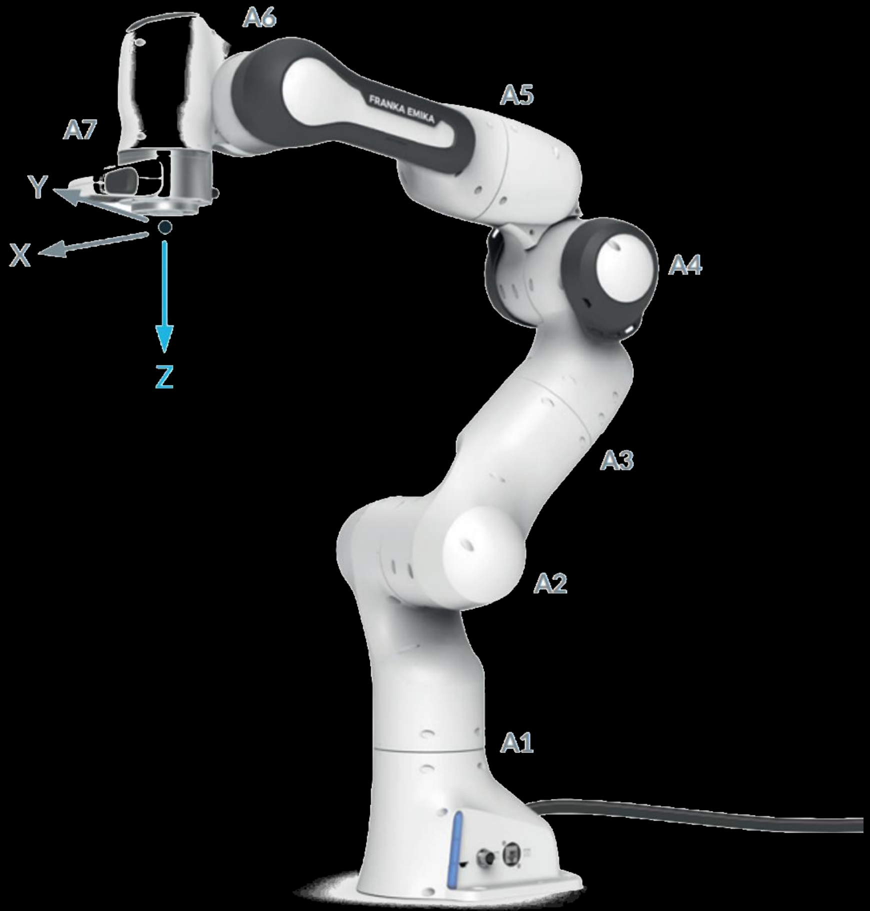
Robotic Arm



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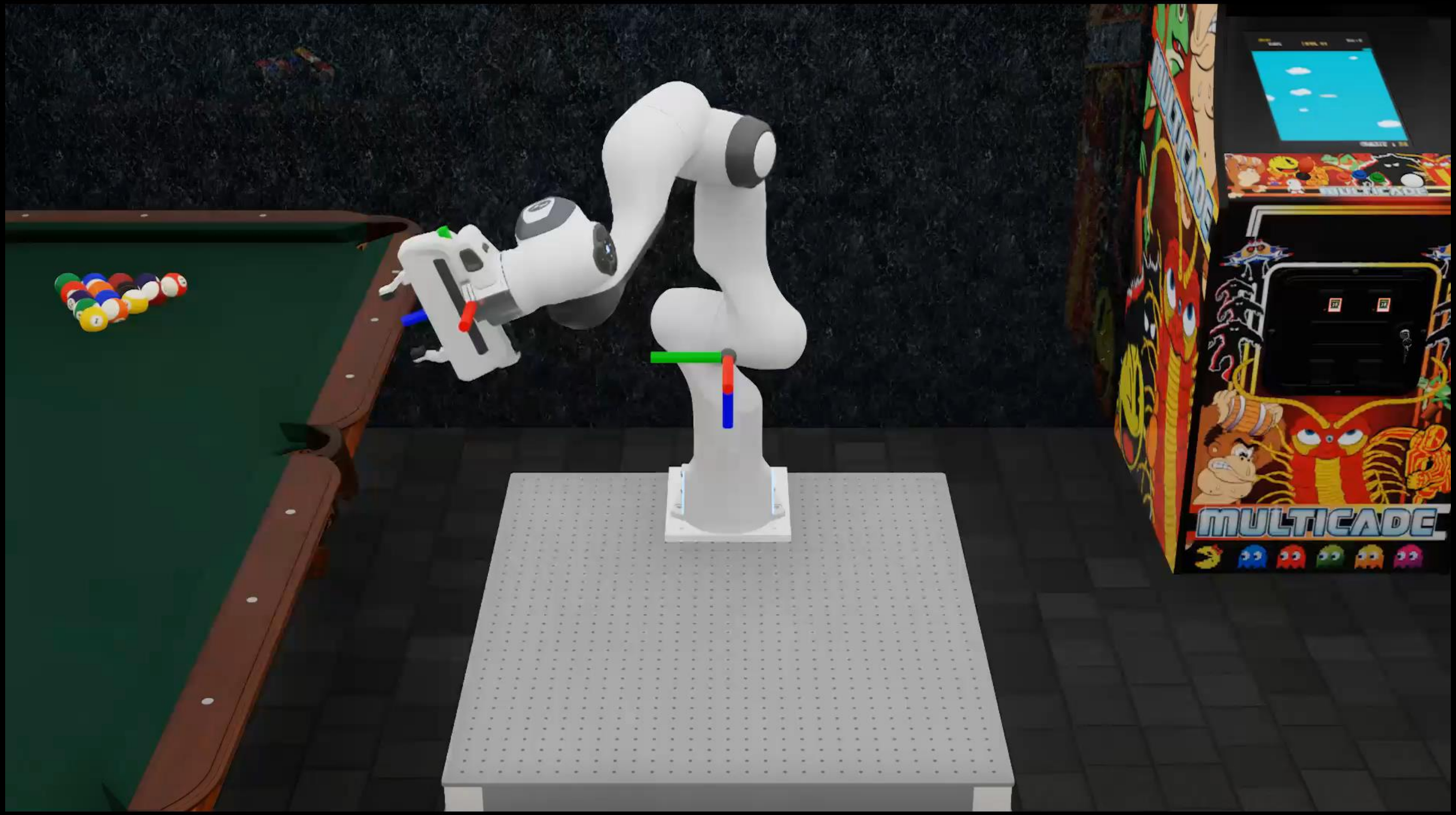

Videos from Orbit

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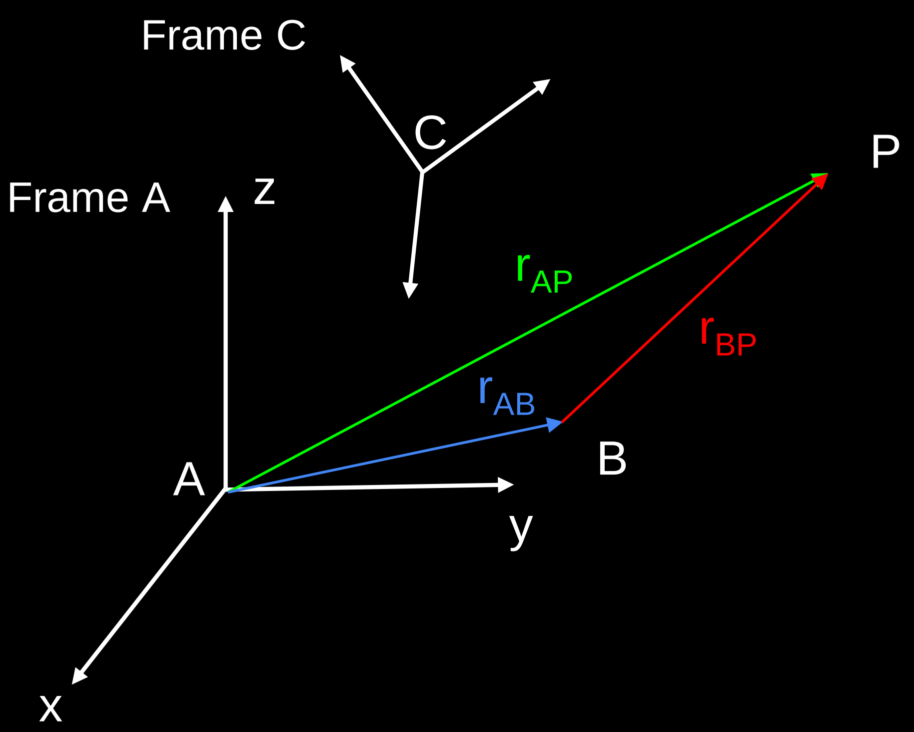
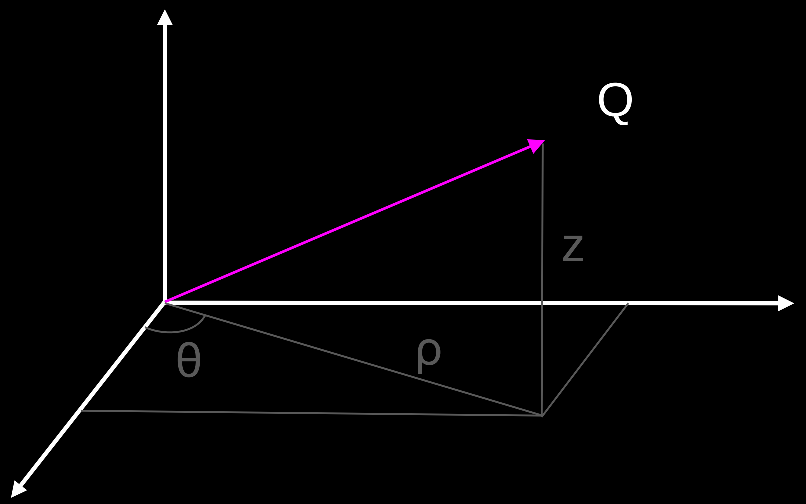
Points, Lines, and Coordinates





Point P in Cartesian Coordinates Frame A: $_{A}X_{P} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$

Point Q in Cylindrical Coordinate: $X_{Q} = \begin{pmatrix} \rho \\ \theta \\ z \end{pmatrix}$

$$_{A}r_{AP} = {_{A}r_{AB}} + {_{A}r_{BP}}$$
$$_{A}r_{AP} \neq {_{A}r_{AB}} + {_{C}r_{BP}}$$

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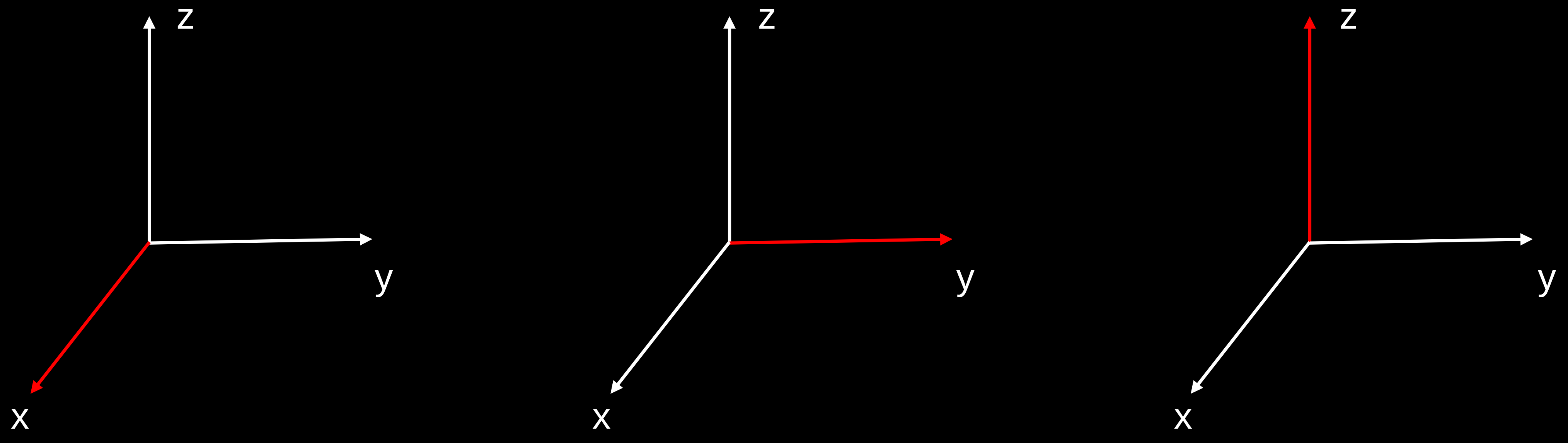
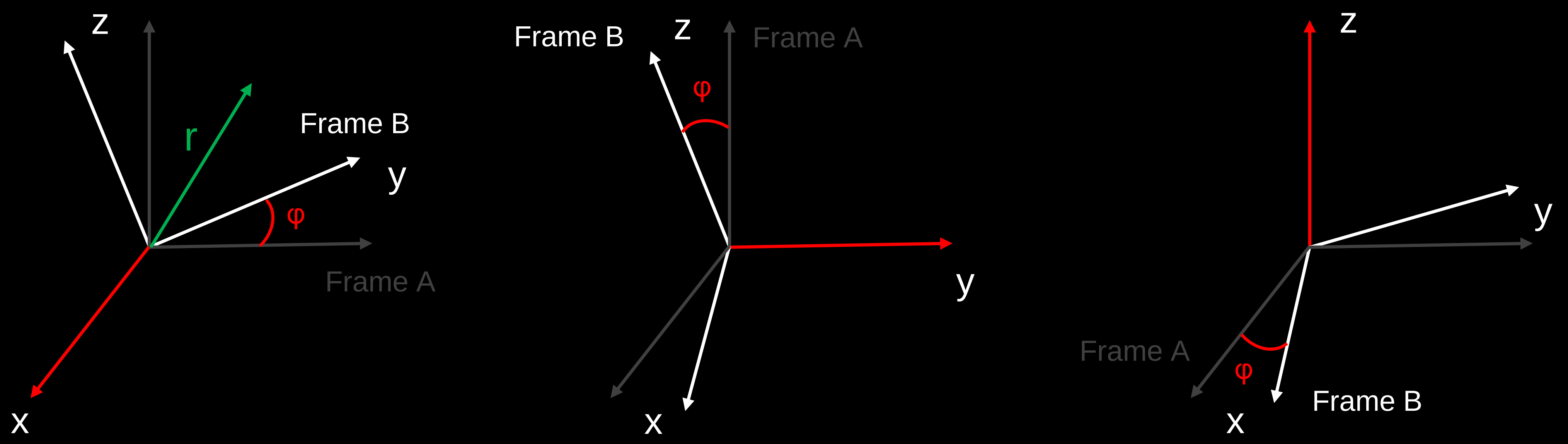
Rotation



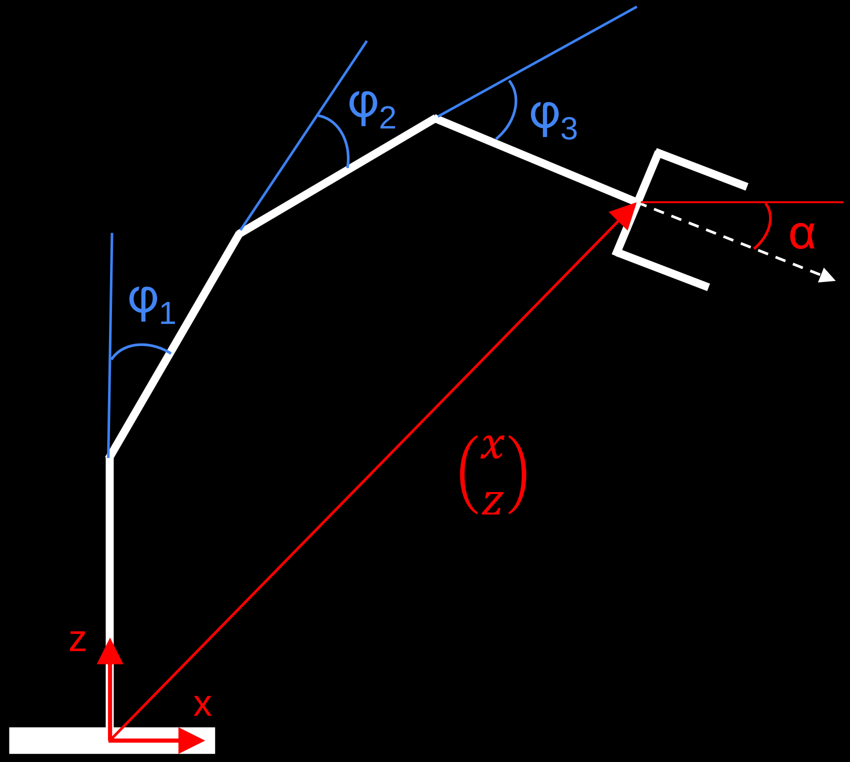
Rotation



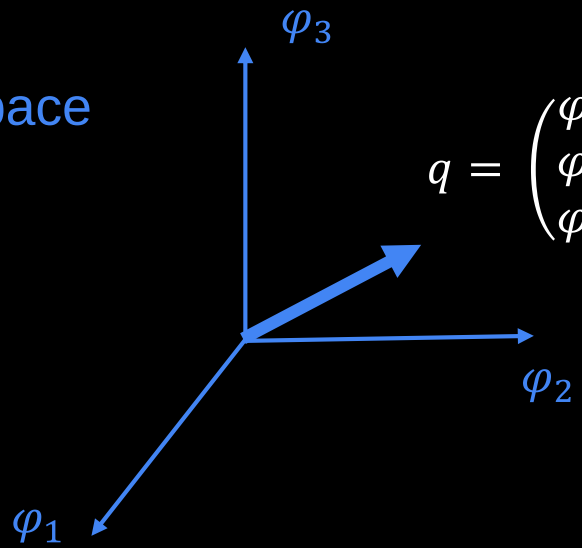
$$C_x(\varphi) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & cos\varphi & -sin\varphi \\ 0 & sin\varphi & cos\varphi \end{bmatrix} \quad C_y(\varphi) = \begin{bmatrix} cos\varphi & 0 & sin\varphi \\ 0 & 1 & 0 \\ -sin\varphi & 0 & cos\varphi \end{bmatrix} \quad C_z(\varphi) = \begin{bmatrix} cos\varphi & -sin\varphi & 0 \\ sin\varphi & cos\varphi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$${}_A r = C_{AB} \cdot {}_B r \rightarrow \begin{pmatrix} {}_A x \\ {}_A y \\ {}_A z \end{pmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & cos\varphi & -sin\varphi \\ 0 & sin\varphi & cos\varphi \end{bmatrix} \begin{pmatrix} {}_B x \\ {}_B y \\ {}_B z \end{pmatrix} = \begin{pmatrix} {}_B x \\ {}_B y \cdot cos\varphi - {}_B z \cdot sin\varphi \\ {}_B y \cdot sin\varphi + {}_B z \cdot cos\varphi \end{pmatrix}$$

Joint Space and Task Space



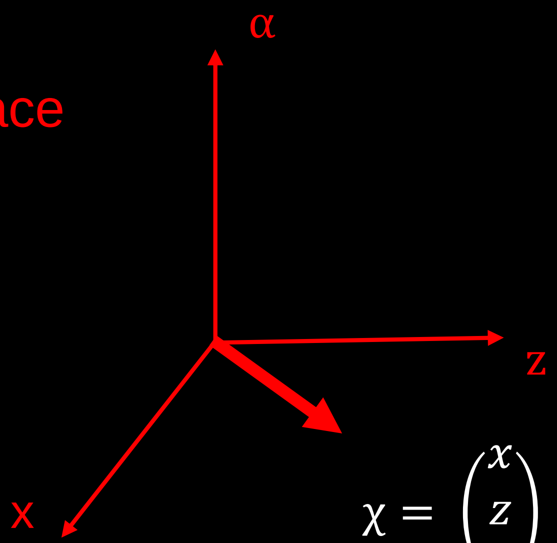
Joint Space



$$q = \begin{pmatrix} \varphi_1 \\ \varphi_2 \\ \varphi_3 \end{pmatrix}$$

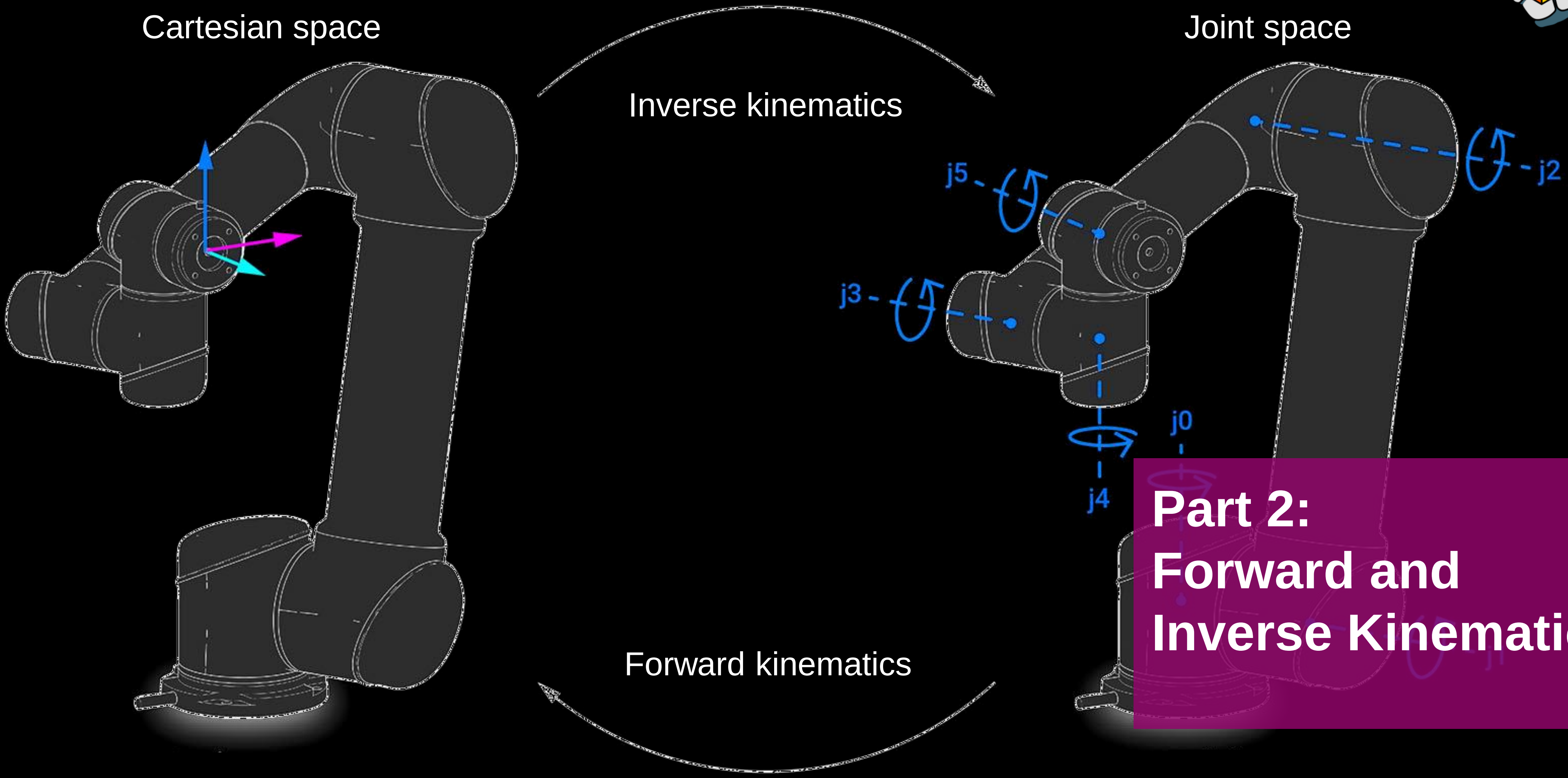
Joint Coordinates

Task Space



$$\chi = \begin{pmatrix} x \\ z \\ \alpha \end{pmatrix}$$

Task Coordinates



Part 2: Forward and Inverse Kinematics

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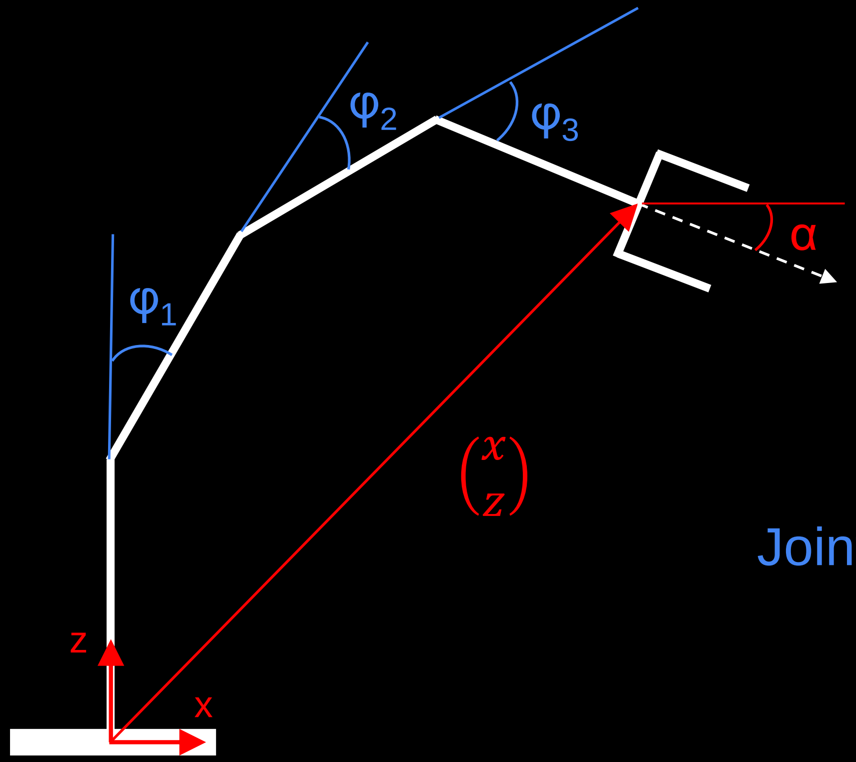
Forward and Inverse Kinematics

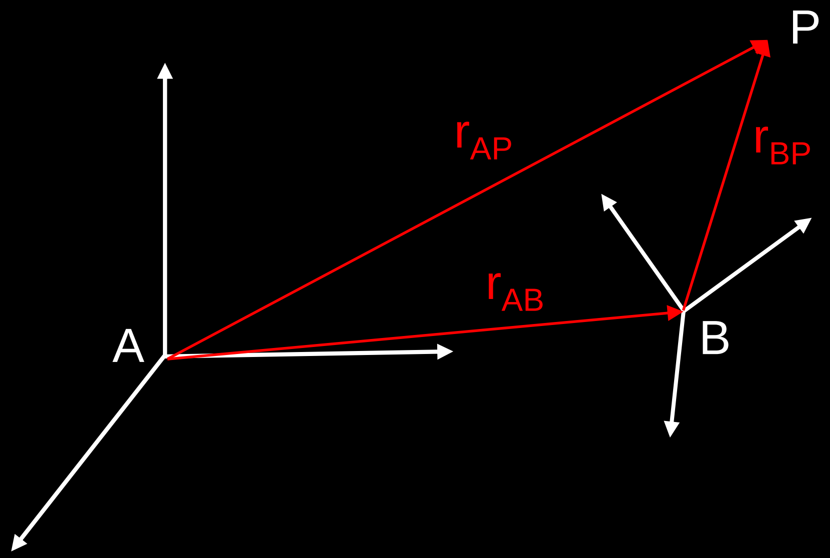


Joint Space → Forward Kinematics → Task Space

Task Space → Inverse Kinematics → Joint Space

From greek ***kinema*** = motion

Homogeneous Transformation Matrix



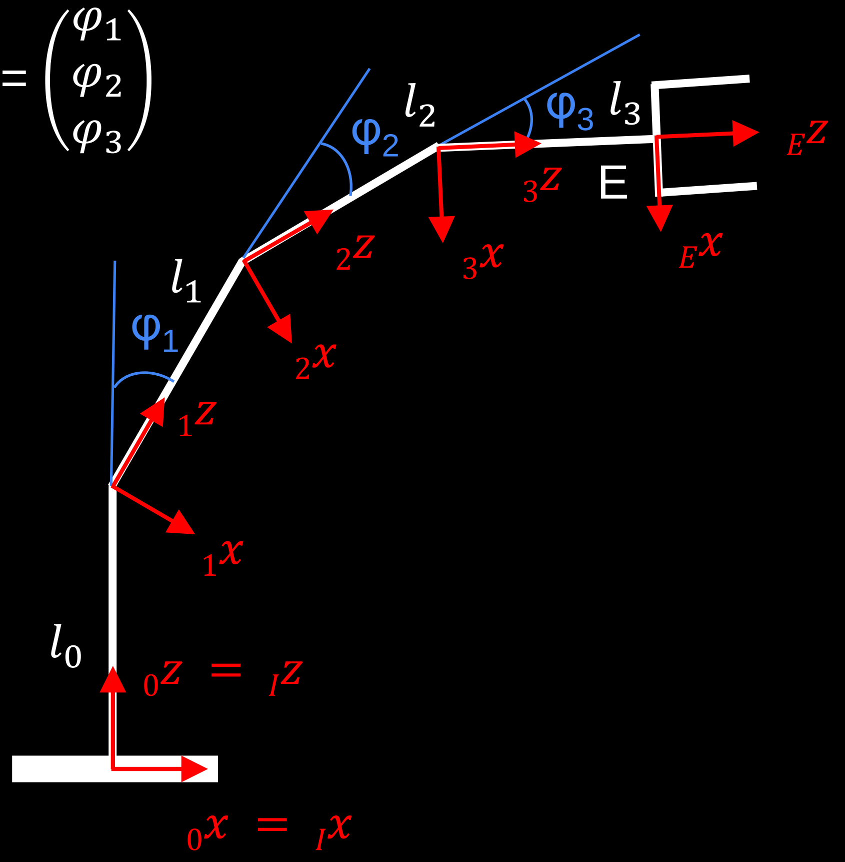
$$r_{AP} = r_{AB} + r_{BP}$$

$${}_A r_{AP} = {}_A r_{AB} + {}_A r_{BP} = {}_A r_{AB} + C_{AB} \cdot {}_B r_{BP}$$

$$\begin{pmatrix} {}_A r_{AP} \\ 1 \end{pmatrix} = \underbrace{\begin{bmatrix} \boxed{C_{AB}} & \boxed{{}_A r_{AB}} \\ 0_{1x3} & 1 \end{bmatrix}}_{T_{AB}} \begin{pmatrix} {}_B r_{BP} \\ 1 \end{pmatrix}$$

Homogeneous Transformation Matrix

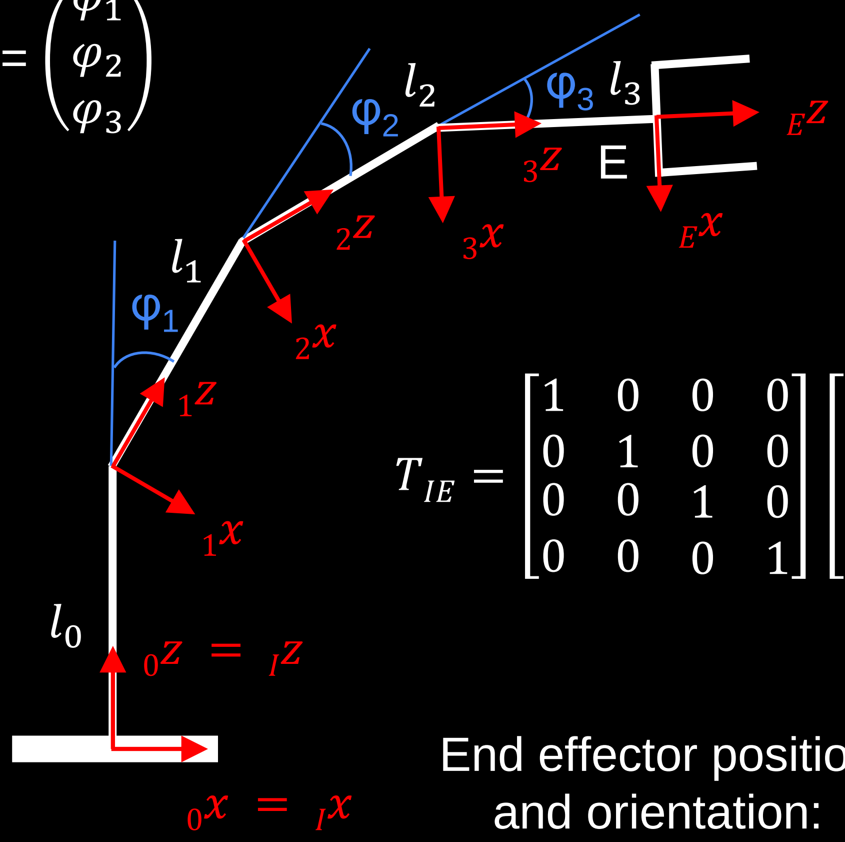
$q = \begin{pmatrix} \varphi_1 \\ \varphi_2 \\ \varphi_3 \end{pmatrix}$



$$T_{IE} = T_{I0} \cdot T_{01} \cdot T_{12} \cdot T_{23} \cdot T_{3E}$$

Homogeneous Transformation Matrix

$$q = \begin{pmatrix} \varphi_1 \\ \varphi_2 \\ \varphi_3 \end{pmatrix}$$



$$T_{IE} = TI_0 \cdot T_{01} \cdot T_{12} \cdot T_{23} \cdot T_{3E}$$

$$T_{IE} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c_1 & 0 & s_1 & 0 \\ 0 & 1 & 0 & 0 \\ -s_1 & 0 & c_1 & l_0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c_2 & 0 & s_2 & 0 \\ 0 & 1 & 0 & 0 \\ -s_2 & 0 & c_2 & l_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c_3 & 0 & s_3 & 0 \\ 0 & 1 & 0 & 0 \\ -s_3 & 0 & c_3 & l_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & l_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

End effector position and orientation:

$${}_IX_E(q) = \begin{pmatrix} l_1 \sin(\varphi_1) + l_2 \sin(\varphi_1 + \varphi_2) + l_3 \sin(\varphi_1 + \varphi_2 + \varphi_3) \\ l_0 + l_1 \cos(\varphi_1) + l_2 \cos(\varphi_1 + \varphi_2) + l_3 \cos(\varphi_1 + \varphi_2 + \varphi_3) \\ \varphi_1 + \varphi_2 + \varphi_3 \end{pmatrix}$$

Forward Differential Kinematics and Jacobian

$$\delta X_E \approx \frac{\delta X_E(q)}{\delta q}\delta q = J_{EA}(q)\delta q$$

with $J_{EA} = \frac{\delta X_E}{\delta q} = \begin{bmatrix} \frac{\delta X_1}{\delta q_1} & \cdots & \frac{\delta X_1}{\delta q_n} \\ \vdots & \ddots & \vdots \\ \frac{\delta X_m}{\delta q_1} & \cdots & \frac{\delta X_m}{\delta q_n} \end{bmatrix}$

$$\dot{X}_E = J_{EA}(q)\dot{q}$$

with $J_{EA}(q) \in \mathbb{R}^{m\times n}$

Inverse Kinematics

- Previously we showed that:

$$J(q)\dot{q} = \chi_e = \begin{bmatrix} \dot{p}_e \\ w_e \end{bmatrix}$$

- If we invert it we obtain:

$$\dot{q} = J^{+}\chi_e \text{ with } \mathrm{J}^{+} = J^{T}\left(JJ^{T}\right)^{-1}$$

- And in a differential form:

$$\Delta\chi_e = J^{+}\Delta q$$

Algorithm 1 Numerical Inverse Kinematics

$\mathbf{q} \leftarrow \mathbf{q}^0$ ▷ Start configuration
while $\|\chi_e^* - \chi_e(\mathbf{q})\| > tol$ **do** ▷ While the solution is not reached
    $\mathbf{J}_{eA} \leftarrow \mathbf{J}_{eA}(\mathbf{q}) = \frac{\partial \chi_e}{\partial \mathbf{q}}(\mathbf{q})$ ▷ Evaluate Jacobian for $\mathbf{q}$
    $\mathbf{J}_{eA}^{+} \leftarrow (\mathbf{J}_{eA})^{+}$ ▷ Calculate the pseudo inverse
    $\Delta\chi_e \leftarrow \chi_e^* - \chi_e(\mathbf{q})$ ▷ Find the end-effector configuration error vector
    $\mathbf{q} \leftarrow \mathbf{q} + \mathbf{J}_{eA}^{+}\Delta\chi_e$ ▷ Update the generalized coordinates
end while

A possible inverse kinematics algorithm

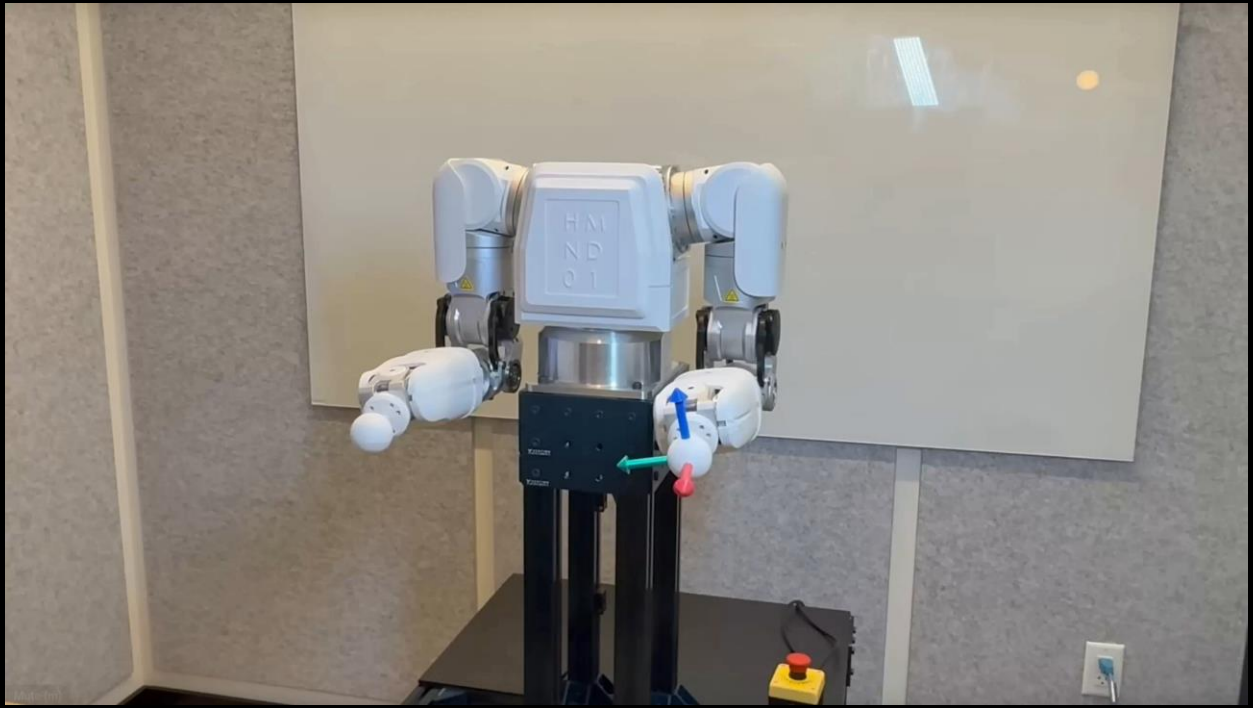
Robot Dynamics Class @ ETH Zurich

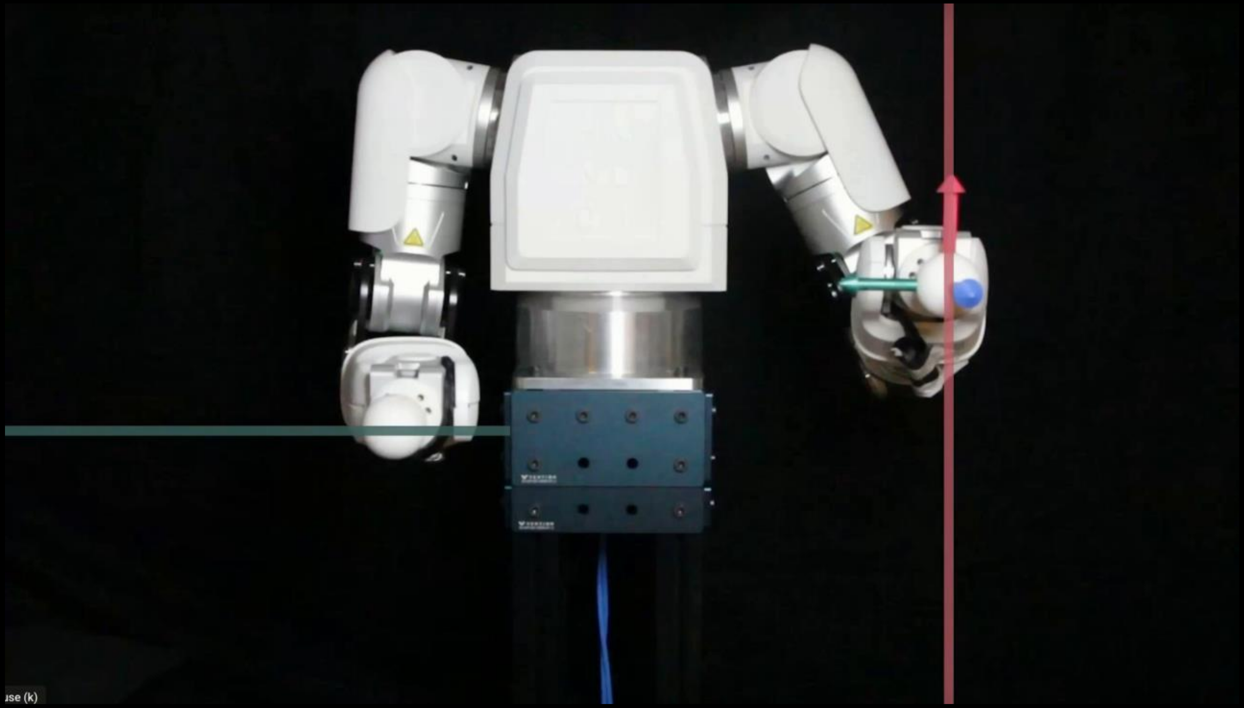
To overcome stability issues, the update can be scaled by a factor k

-> slower convergence

$$q \leftarrow q + kJ_{eA}^{+}\Delta\chi_e \text{ with } k \in (0,1)$$

Inverse Kinematics





https://thehumanoid.ai/

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Part 3:
Kinematics and Dynamics for Hand Joints

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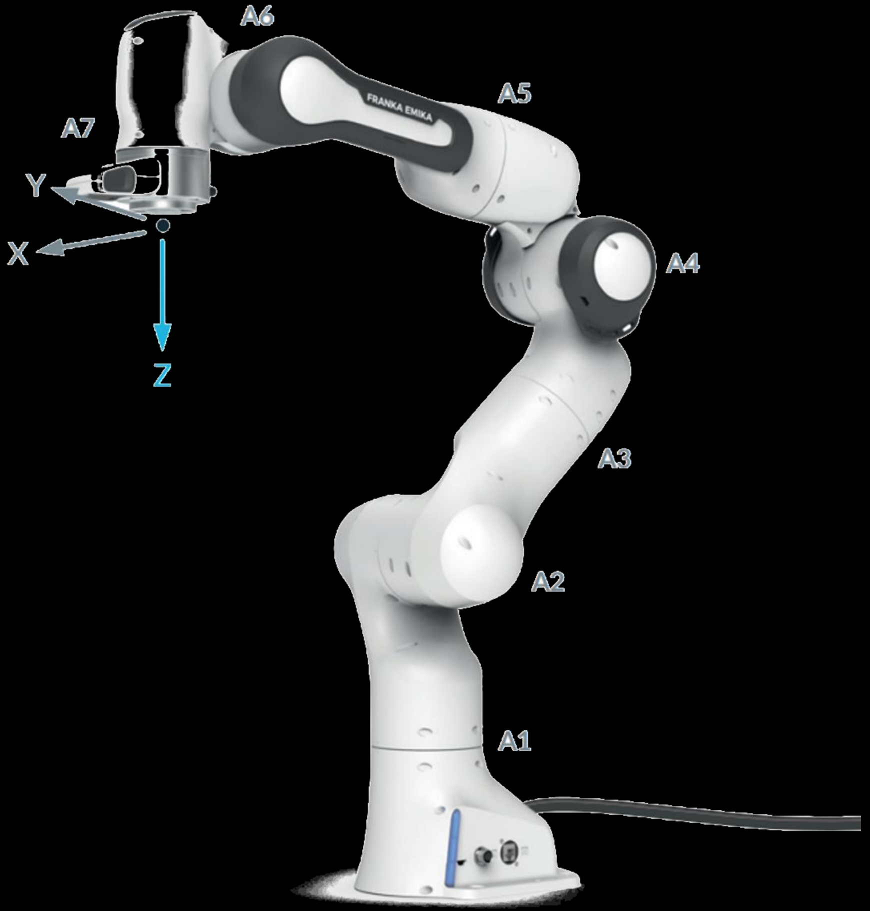
Difference Between Conventional Robots and Robotic Hands



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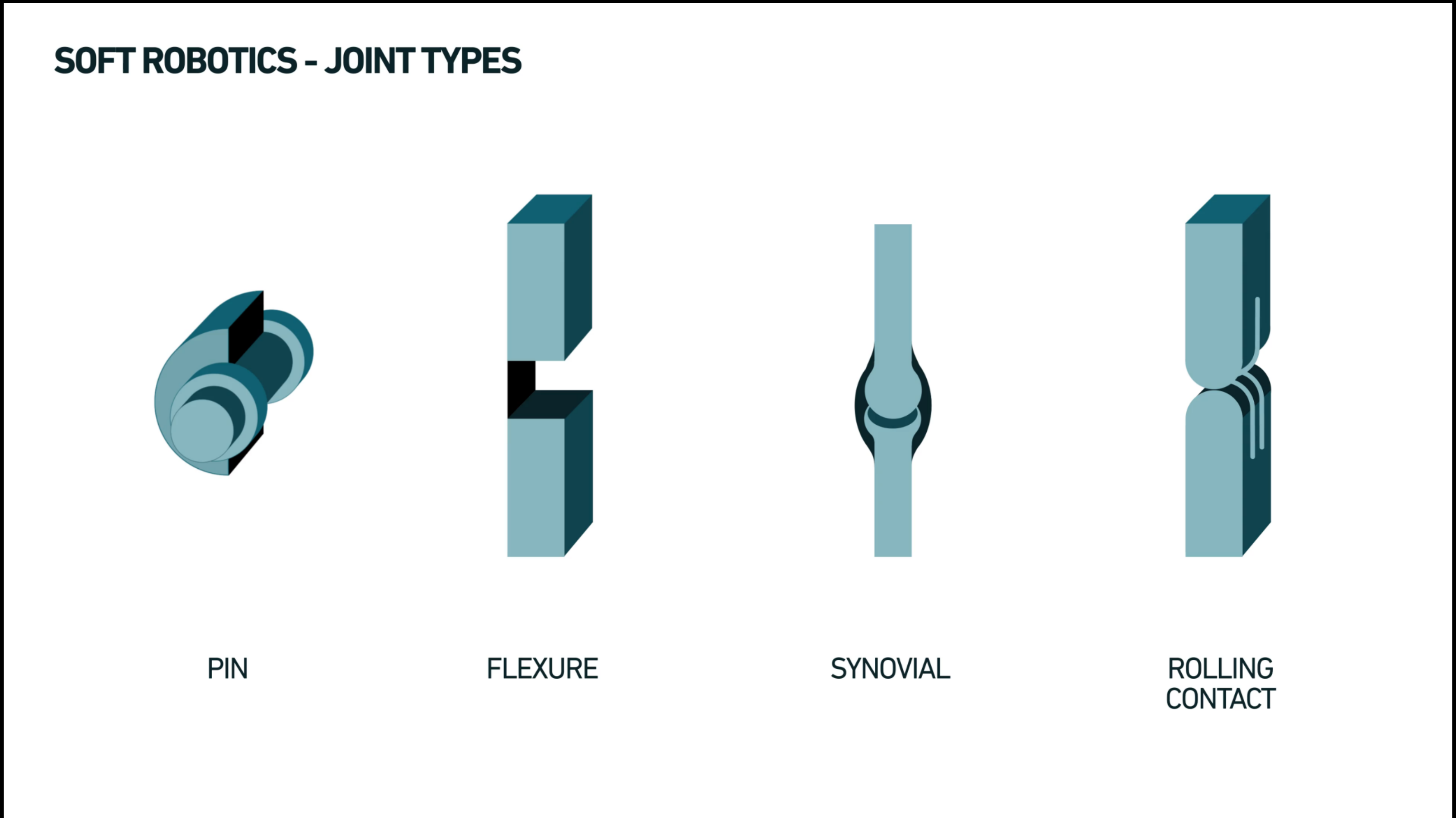
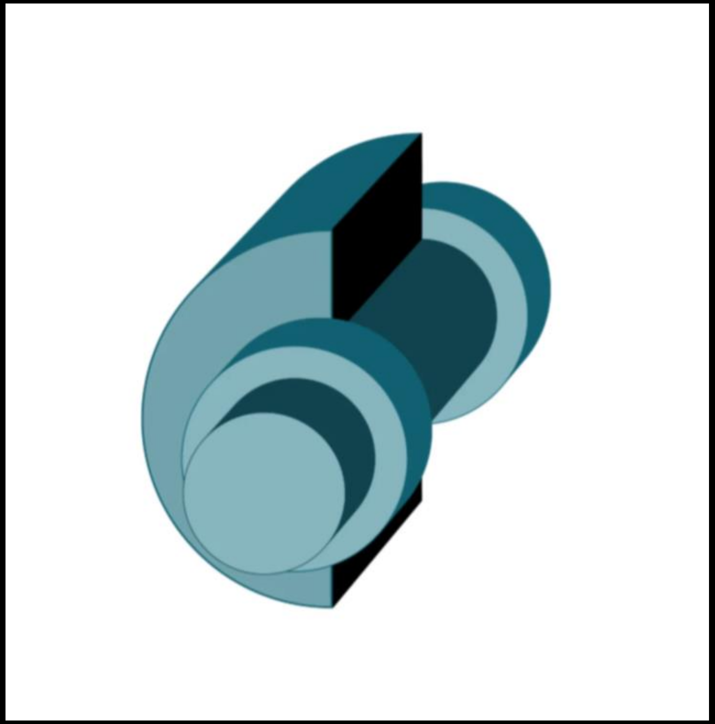
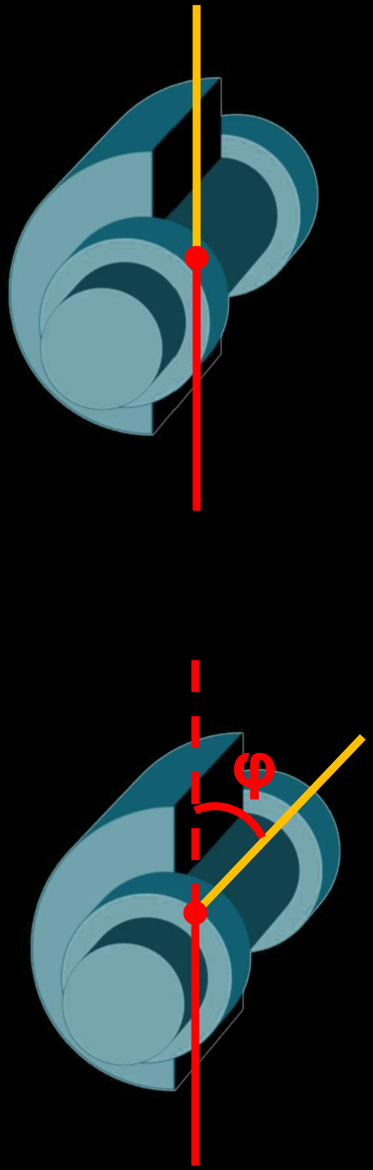
Recap: Different Types of Joints



Pin Joint

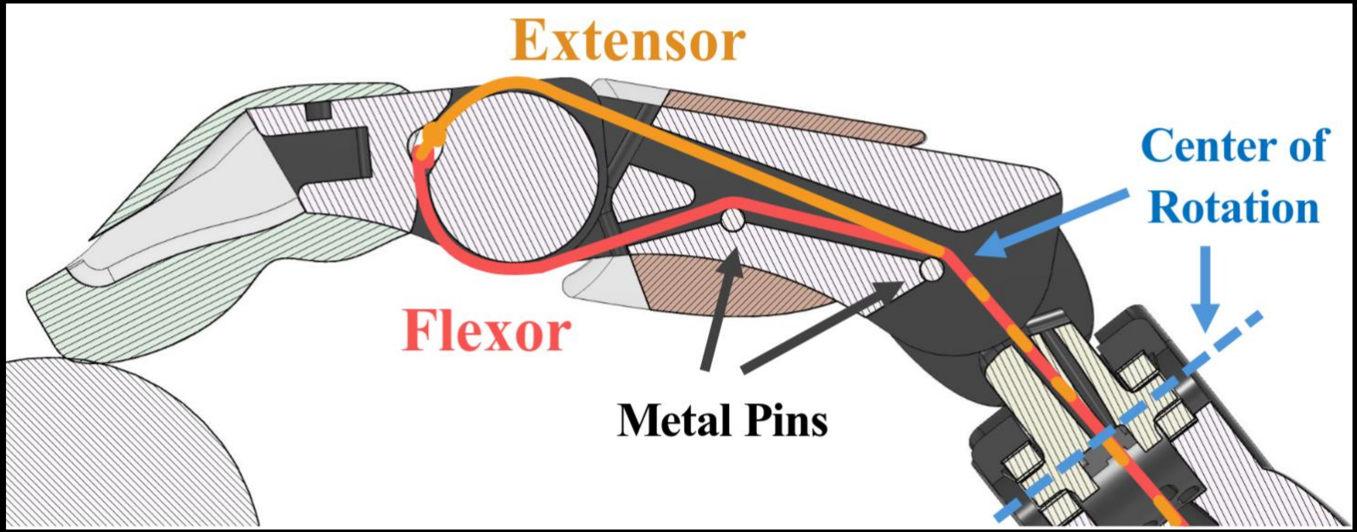
What the ORCA Hand uses



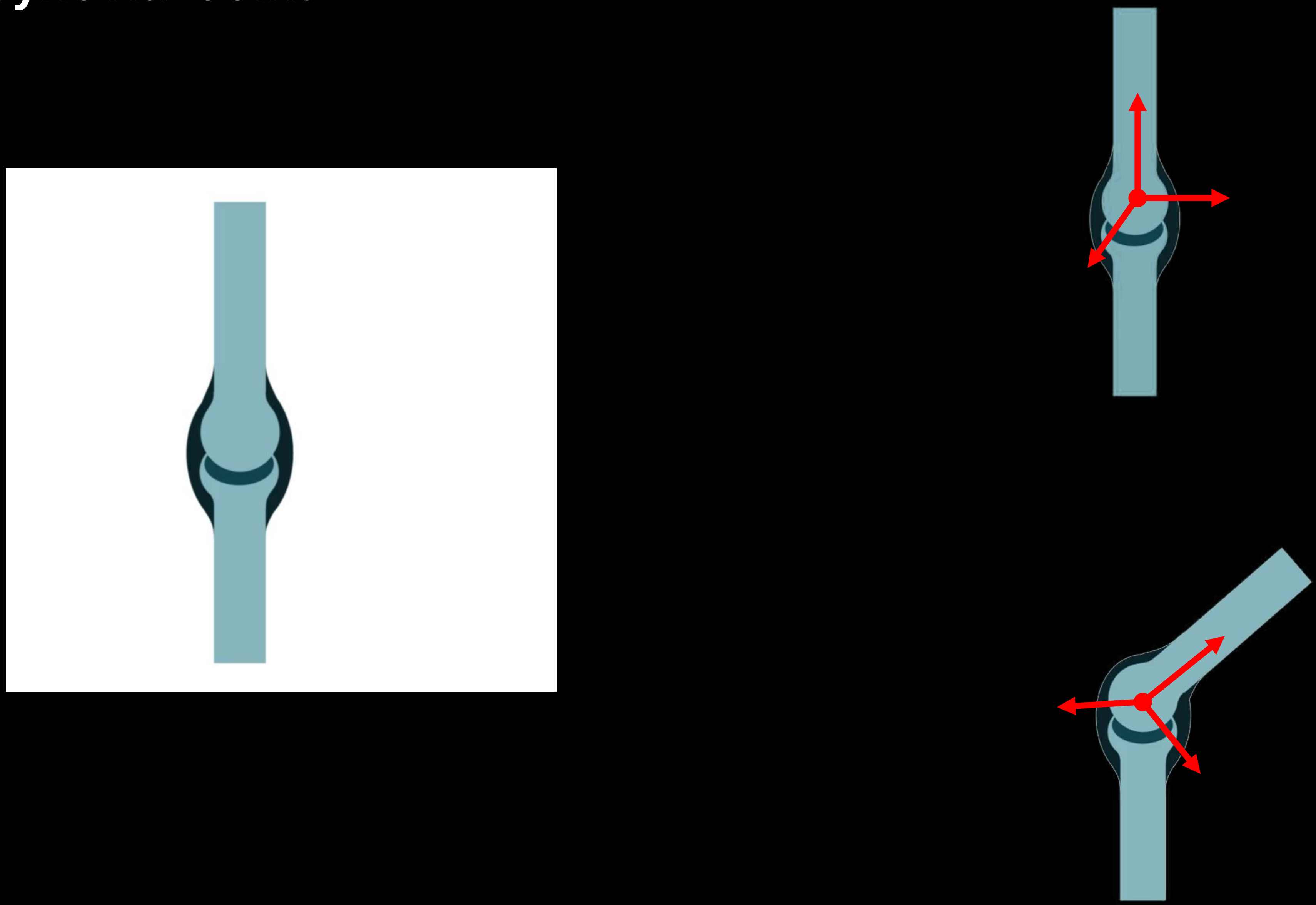


Pin Joint

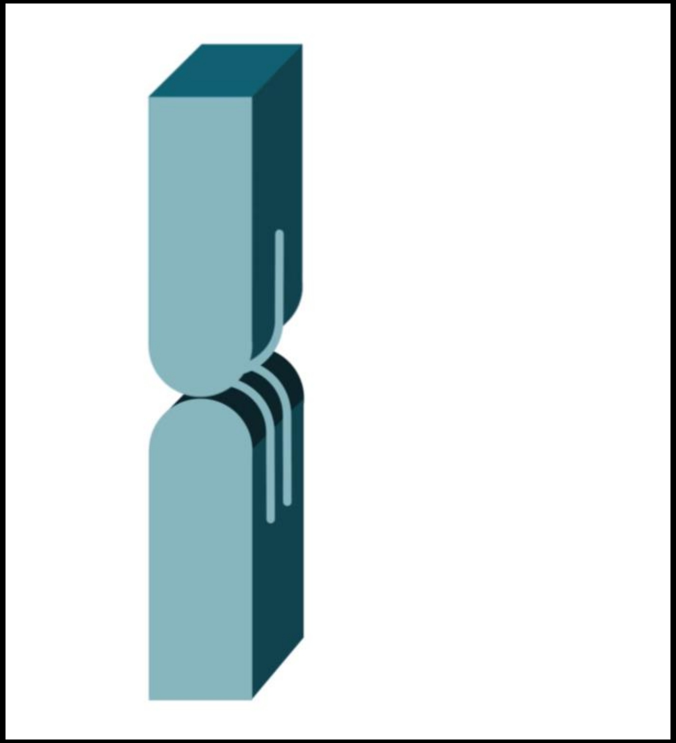
What the ORCA Hand uses

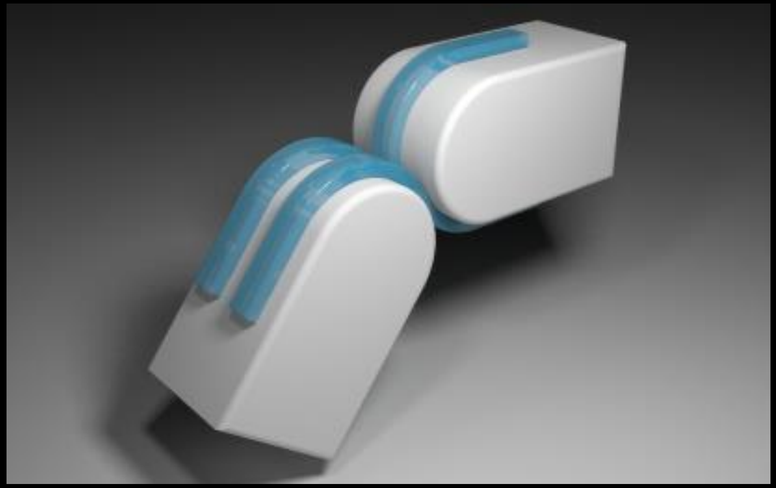


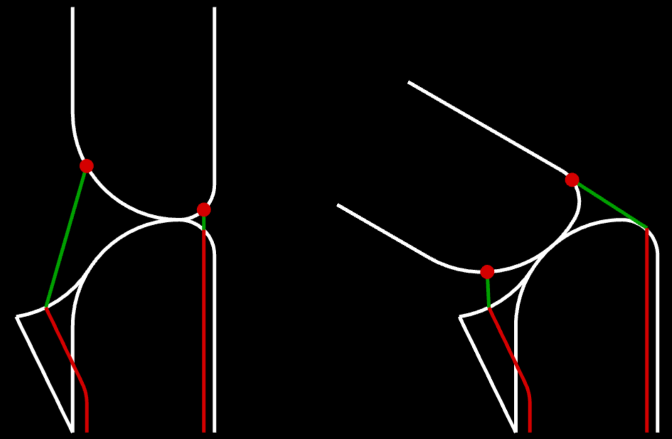
Synovial Joint



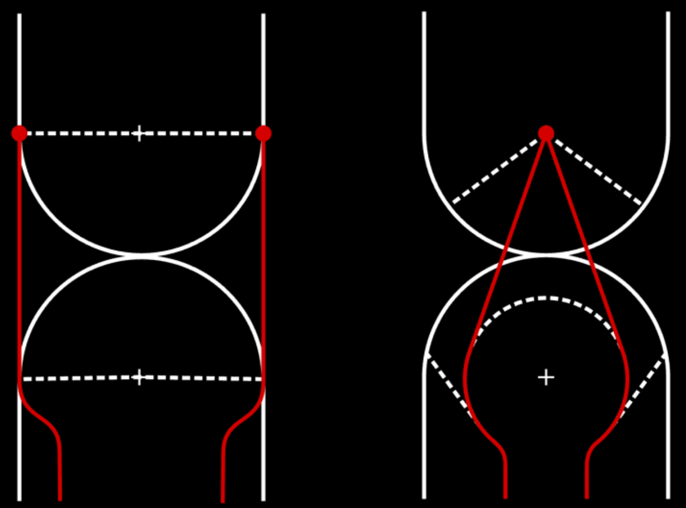
Rolling Contact Joint – Joints Used on Faive Hand

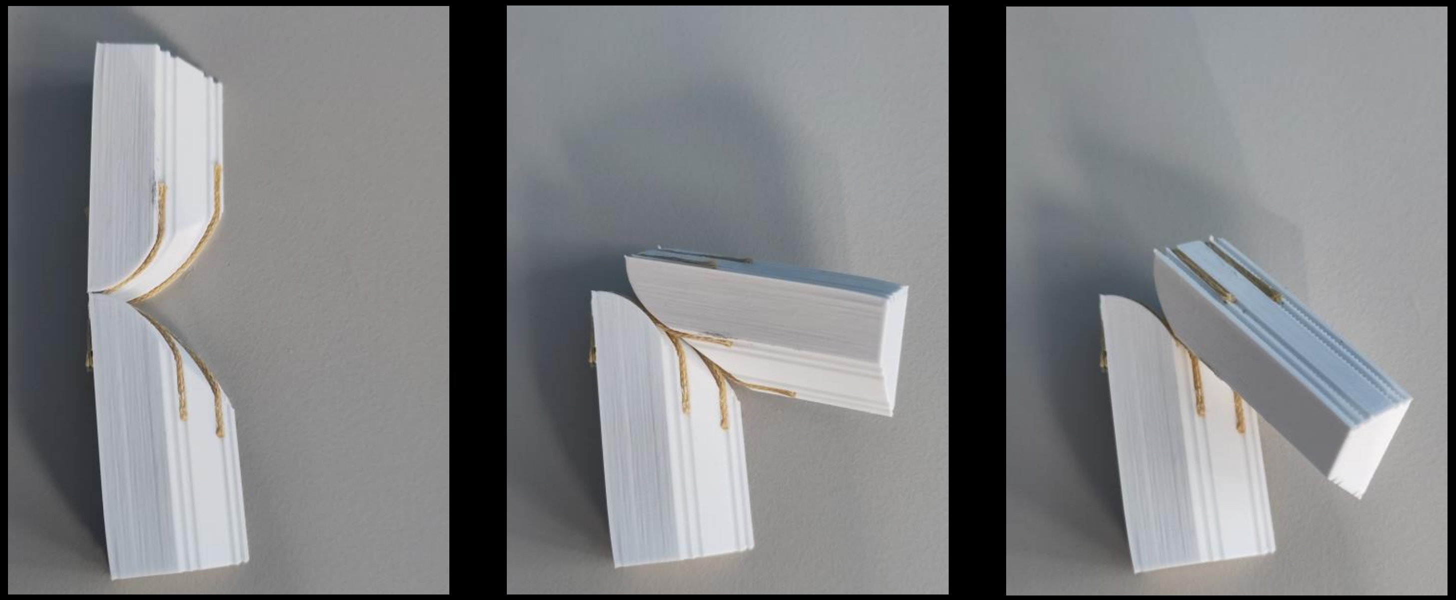




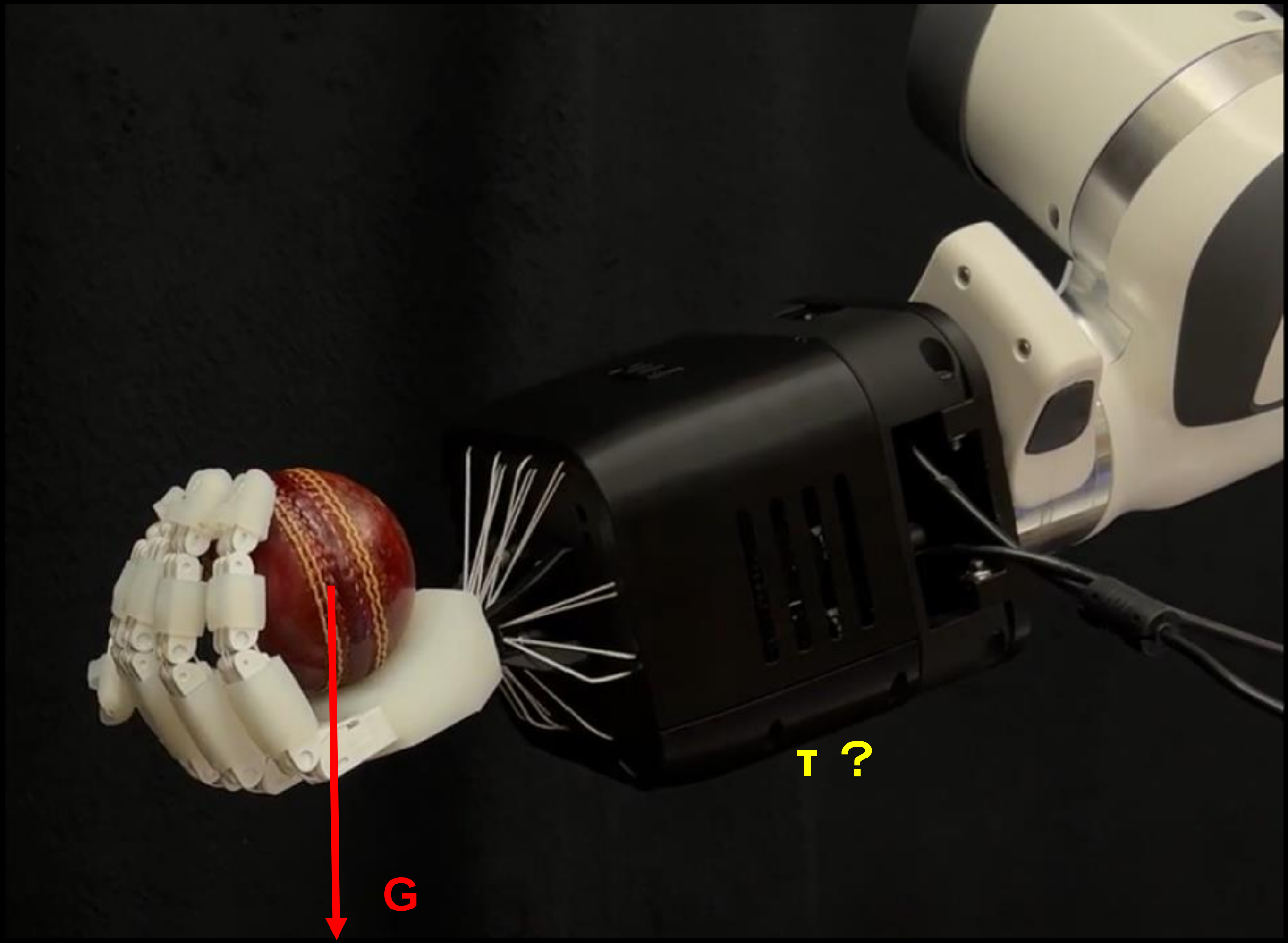


Kinematics for Rolling Contact Joint





Dynamics



Toshimitsu et al. (2023) https://srl-ethz.github.io/get-ball-rolling/

Dynamics

Tendon Lengths

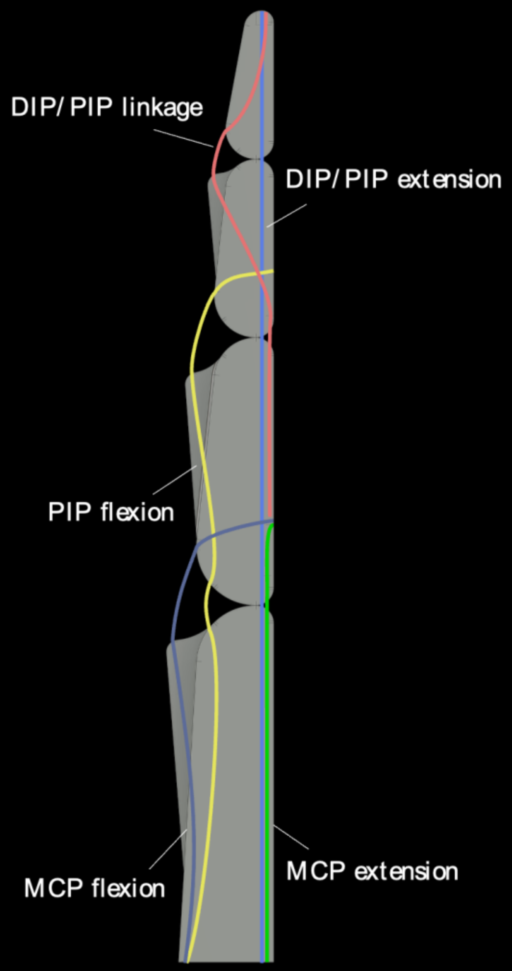
$$p = g(l) = g\big(f(q)\big) = F(q)$$

Motor Positions

Joint Angles

Dynamics – Jacobian

$$J_m = \begin{bmatrix} \frac{\partial p_1}{\partial q_1} & \frac{\partial p_1}{\partial q_2} \\ \frac{\partial p_2}{\partial q_1} & \frac{\partial p_2}{\partial q_2} \end{bmatrix}$$



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Dynamics

Velocity of the finger joints

$$\dot{p} = J_m \cdot \dot{q}$$

Velocity of the motors

$$\tau^T \cdot \dot{q} = T^T \cdot \dot{p}$$

Conservation of Power

$$\tau^T \cdot \dot{q} = T^T \cdot J_m \cdot \dot{q}$$

$$\tau = {J_m}^T \cdot T$$

Dynamics

Previous slide: $\tau = {J_m}^T \cdot T$

$$\dot{X}_{fingertip} = J_{fingertip} \cdot \dot{q}$$

$$\tau^T \cdot \dot{q} = {F_{fingertip}}^T \cdot \dot{X}_{fingertip}$$

$$\tau^T \cdot \dot{q} = {F_{fingertip}}^T \cdot J_{fingertip} \cdot \dot{q}$$

$$\tau = {J_{fingertip}}^T \cdot F_{fingertip}$$

Dynamics

$$\tau = {J_m}^T \cdot T$$

$$\tau = {J_{fingertip}}^T \cdot F_{fingertip}$$

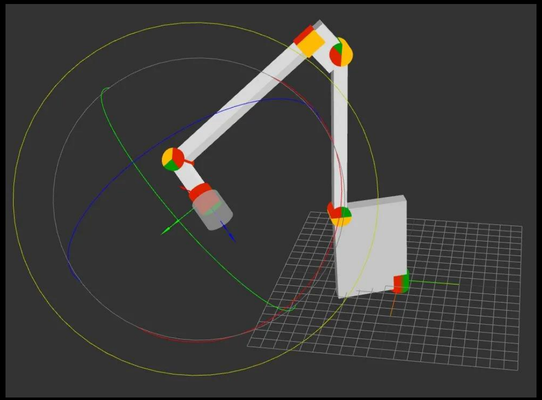
$$T = \left({J_m}^T\right)^{-1} \cdot {J_{fingertip}}^T \cdot F_{fingertip}$$

Summary of Kinematics and Dynamics

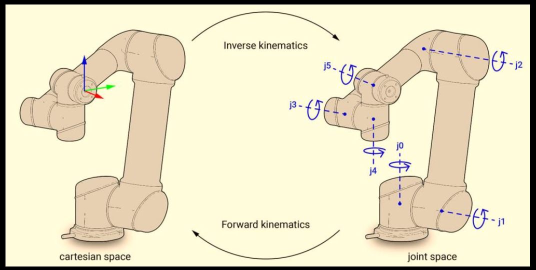
- Intro to Robot Kinematics and Dynamics
  - Representing points and lines in different coordinates and frames
  - Rotational matrix
  - Joint space and task space

- Forward and Inverse Kinematics
  - Homogeneous transformation matrix
  - Forward differential kinematics and Jacobian
  - Inverse kinematics

- Kinematics and Dynamics for hand joints
  - Hand Joints
  - Kinematics for rolling joints
  - Dynamics for rolling joints



Marginally Clever Robots



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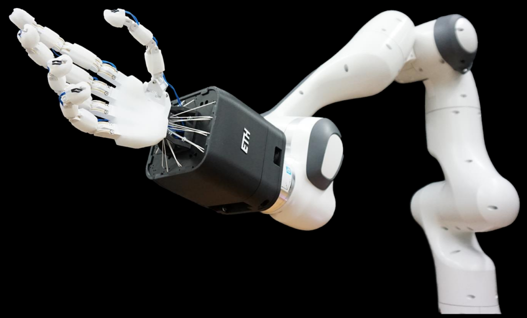

Faive Robotics

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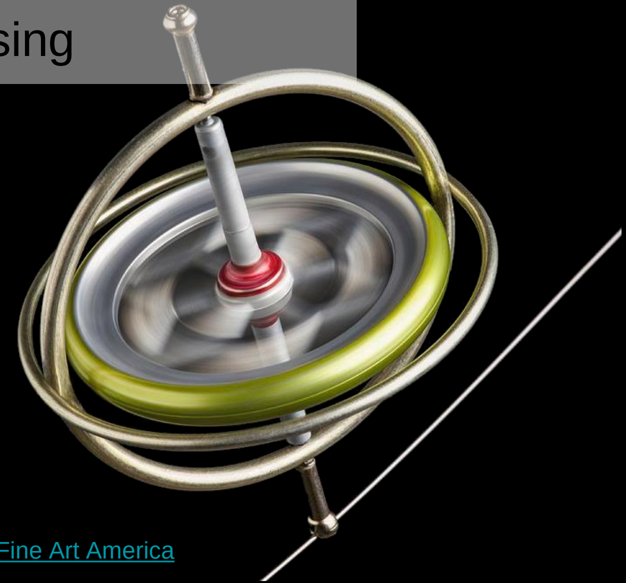
Implementing Control Strategies for Manipulation!

1. Sensing

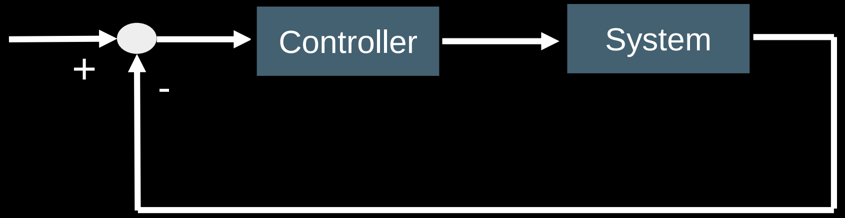


Fine Art America

Next Week

2. Control

Wikimedia



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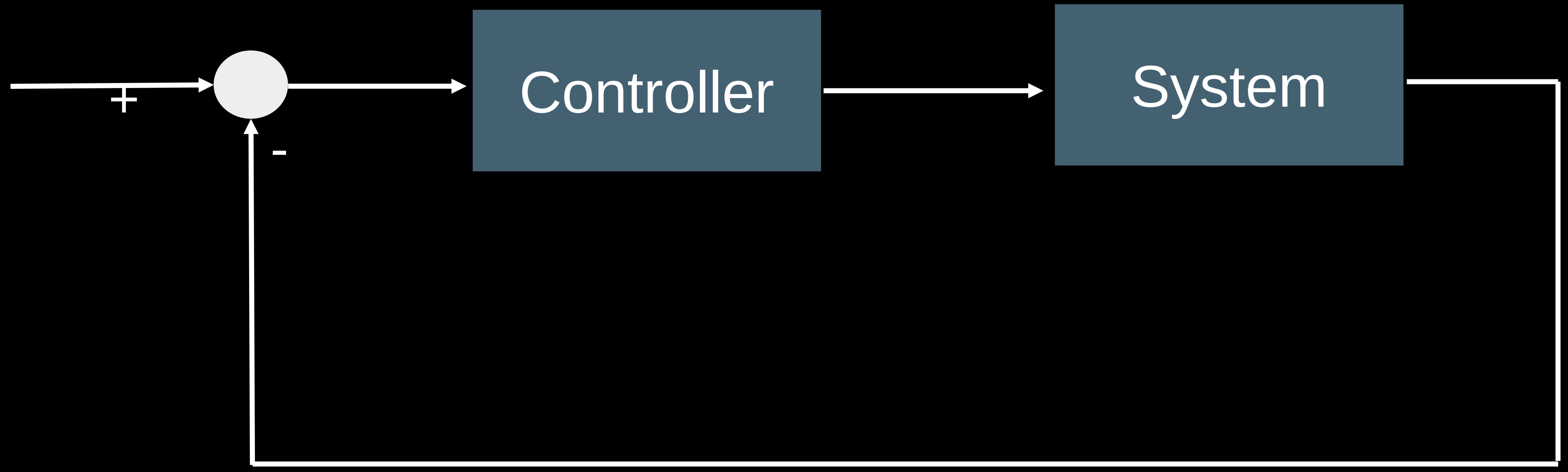

Part 4a: Feedback Control

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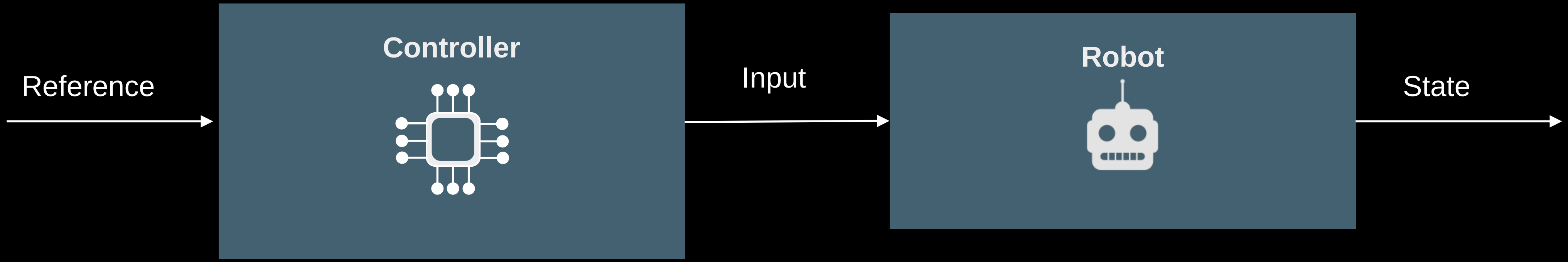
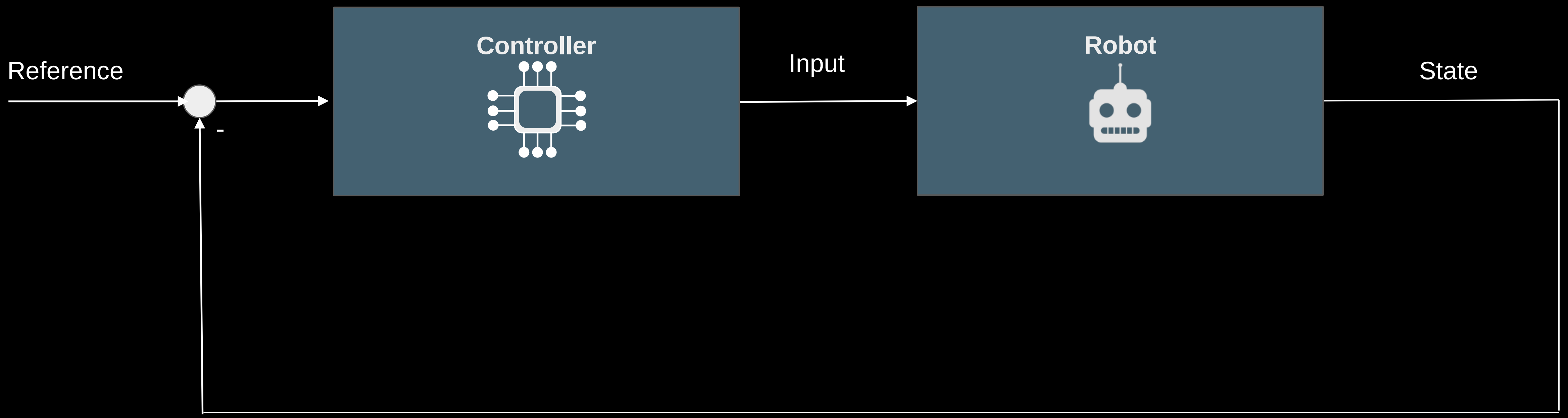
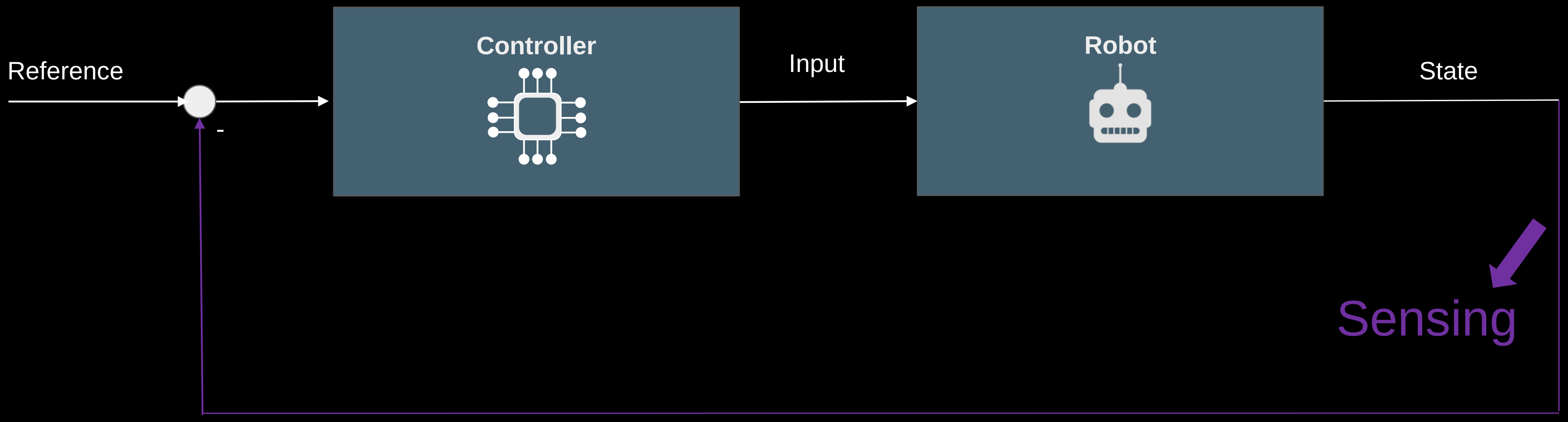
Simplest controller possible: Open loop



Closed Loop Controller



Closed Loop Controller

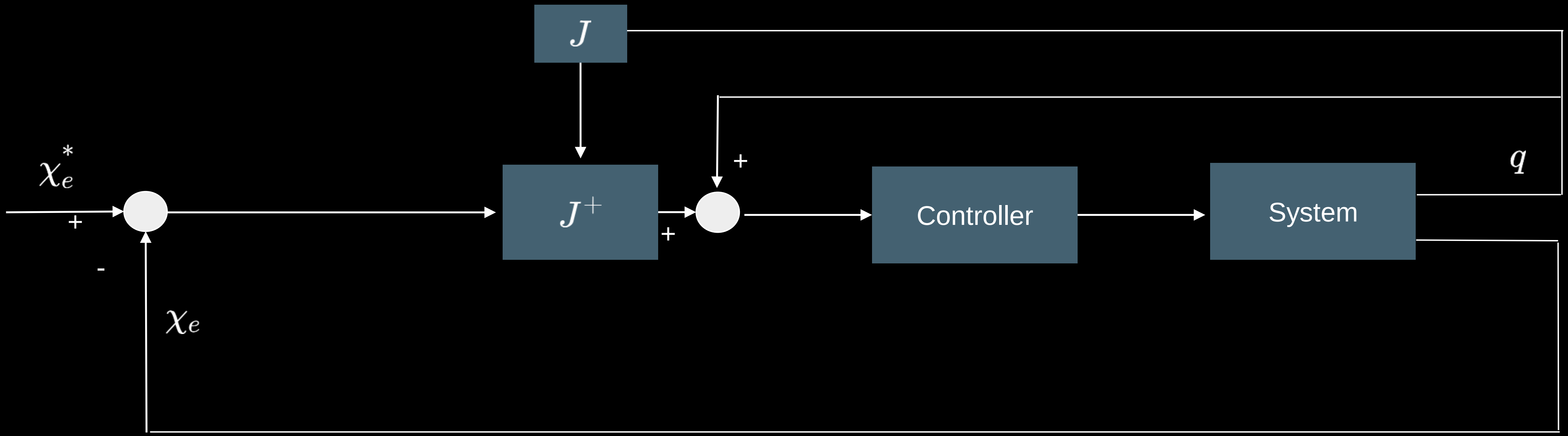


Remember - Inverse Kinematics

Numerical Inverse Kinematics (iterative approach):

$$q \leftarrow q + kJ_{eA}^{+}\Delta\chi_e \text{ with } k \in (0,1)$$

Inverse Kinematics Control



Trajectory Control

We can use a closed loop controller, but we need to add a component for the desired velocities

We define $$\Delta r_e^t = r_e^*(t) - r_e(q^t)$$

And the desired joint velocity $$\dot{q}^* = J_{e0_P}^+(q^t) \cdot (\dot{r}_e^*(t) + k_{pP} \Delta r_e^t)$$

If we have a desired rotation rate we write $$\dot{q}^* = J_{e0_R}^+(q^t) \cdot (\omega_e^*(t) + k_{pR} \Delta \phi)$$

Where ϕ are the angles used to represent the orientation of the end effector.

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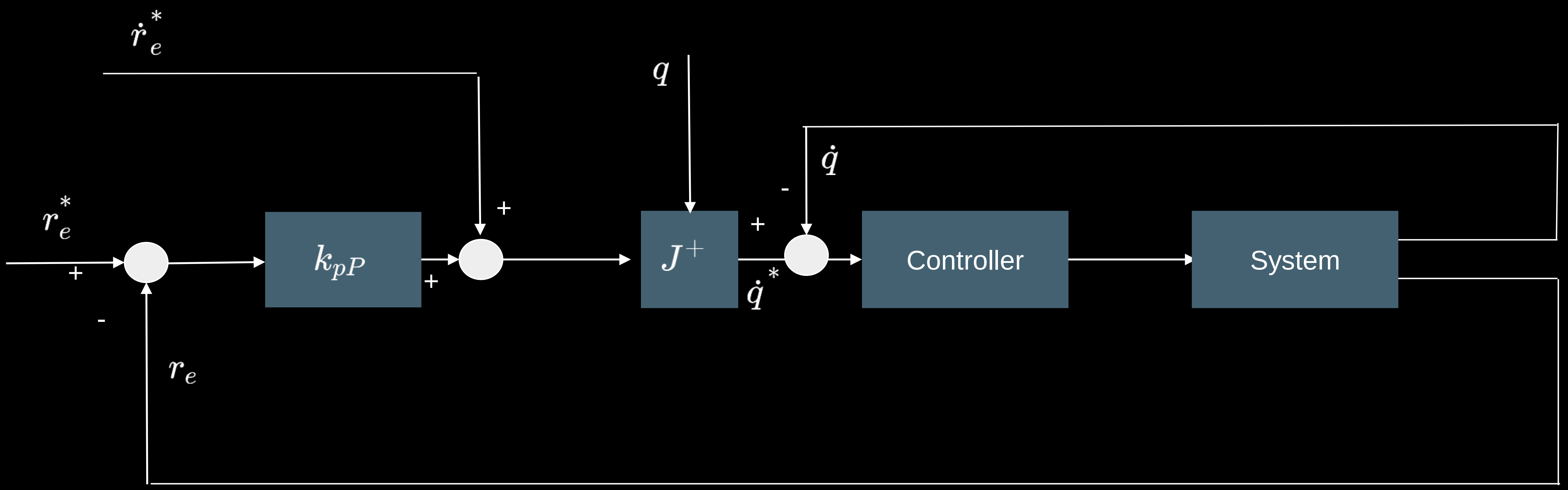
Trajectory Control



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Dynamic control

The dynamic model is

$$M(q)\ddot{q} + b(q,\dot{q}) + g(q) = \tau + J_c(q)^T F_c$$

With:

$M(q)$: Generalized mass matrix

$q, \dot{q}, \ddot{q}$: Generalized position, velocity and acceleration vector

$b(q,\dot{q})$: Coriolis and centrifugal terms

$g(q)$: Gravitational terms

τ : External generalized forces

F_c : External Cartesian forces

$J_c(q)$: Geometric Jacobian corresponding to the external forces

Dynamic control

The dynamic model is

$$M(q)\ddot{q} + b(q,\dot{q}) + g(q) = \tau + J_c(q)^T F_c$$

If we know the desired generalized accelerations, velocities and poses we can write

$$\ddot{q}^* = k_p(q^* - q) + k_d(\dot{q}^* - \dot{q})$$

Thus the joint torques will be

$$\tau^* = M(q)\ddot{q}^* + b(q,\dot{q}) + g(q)$$

Task-space control

Remember that $J(q)\dot{q} = \chi_e = \begin{bmatrix} \dot{p}_e \\ w_e \end{bmatrix}$

If you derive that with respect to time: $\dot{\chi}_e = J(q)\ddot{q} + \dot{J}(q)\dot{q}$

And if we solve the dynamics equation for the joint acceleration and substitute in the equation above we get:

$$\dot{\chi}_e = JM^{-1}(\tau - b - g) + \dot{J}\,\dot{q}$$

Finally, remembering that $\tau = J_e^T F_e$

We can write $$\Lambda_e \dot{\chi}_e + \mu + p = F_e$$

$$\Lambda_e = (J_e M^{-1} J_e^T)^{-1}$$
$$\mu = \Lambda_e J_e M^{-1} b - \Lambda_e \dot{J}_e \dot{q}$$
$$p = \Lambda_e J_e M^{-1} g$$

Task-space control

Defining the dynamics uniquely depending on the state of the end effector allows us to design a control loop

$$\boxed{\dot{\chi}_e^*} = \begin{pmatrix} r_e^* - r_e \\ \Delta\phi_e \end{pmatrix} + k_d(\chi_e^* - \chi_e)$$

Trajectory/Task space control → PID Control

- **Idea:** Compare desired vs actual output → **compute error** → apply correction (Proportional, Integral, Derivative).
- **Strengths:** Simple, cheap, widely used, doesn't require a full model.
- **Weaknesses:** Limited with nonlinear or high-dimensional systems; needs tuning; not predictive.

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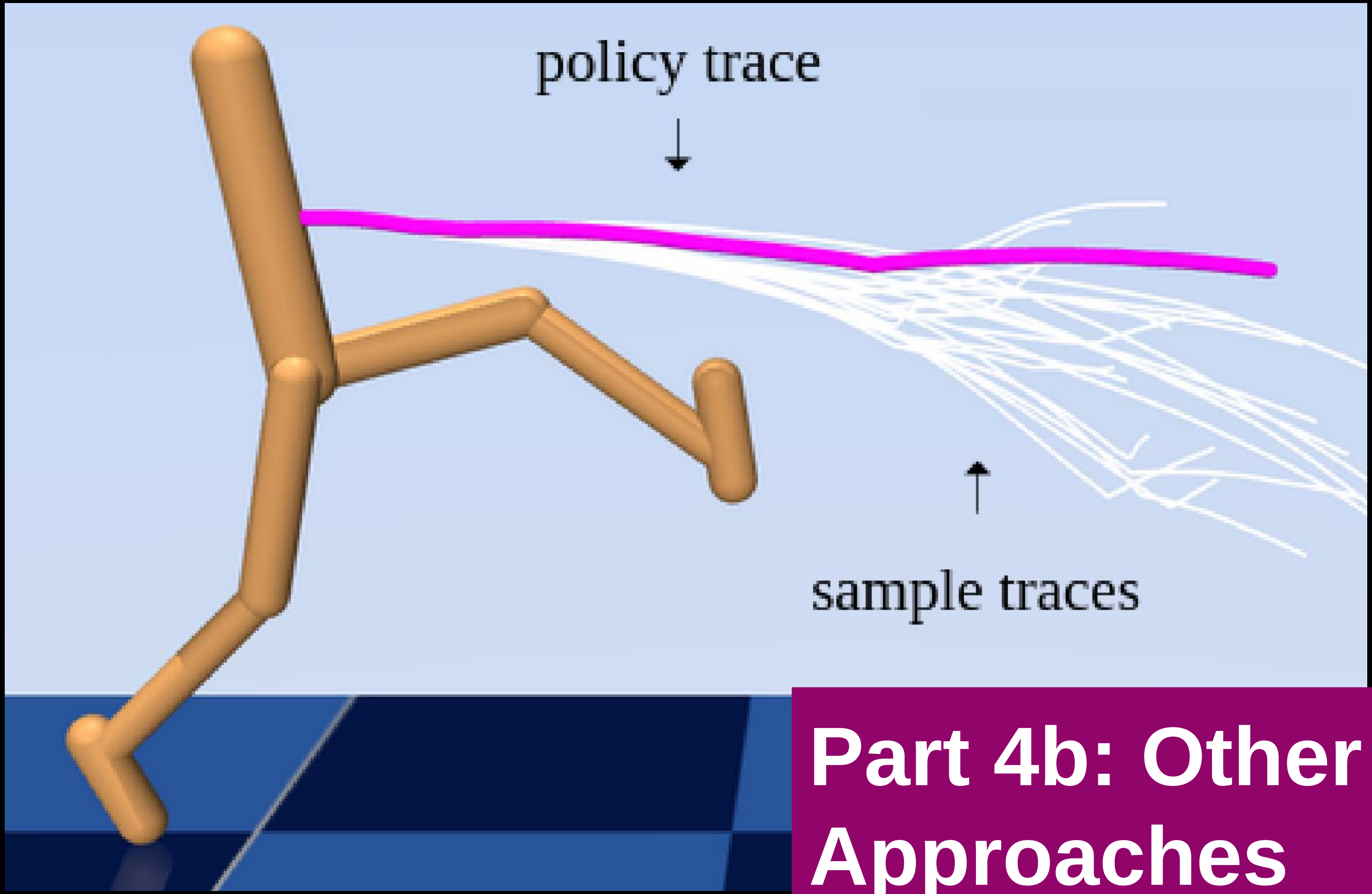

Mujoco PC

Part 4b: Other Control Approaches

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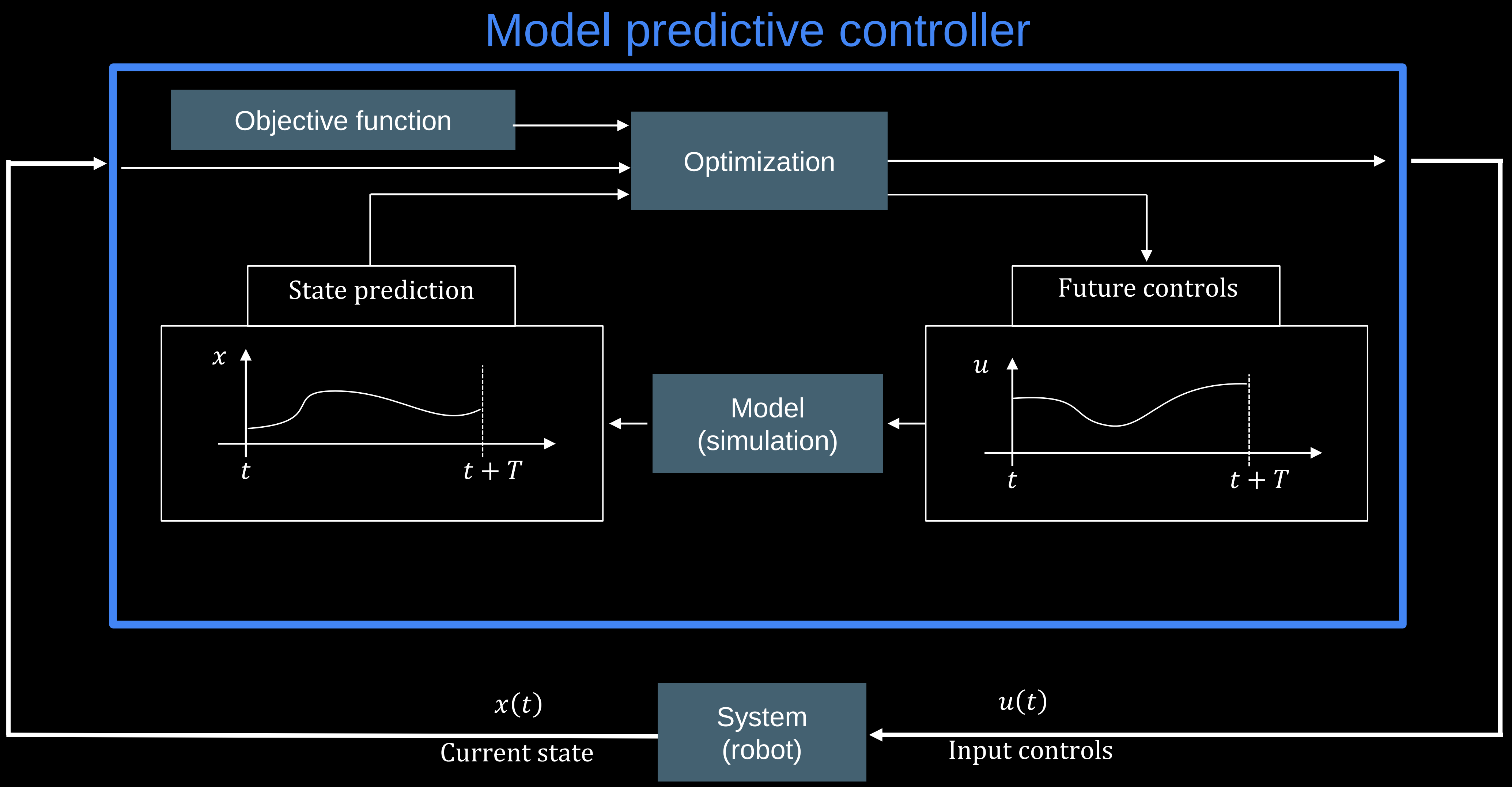
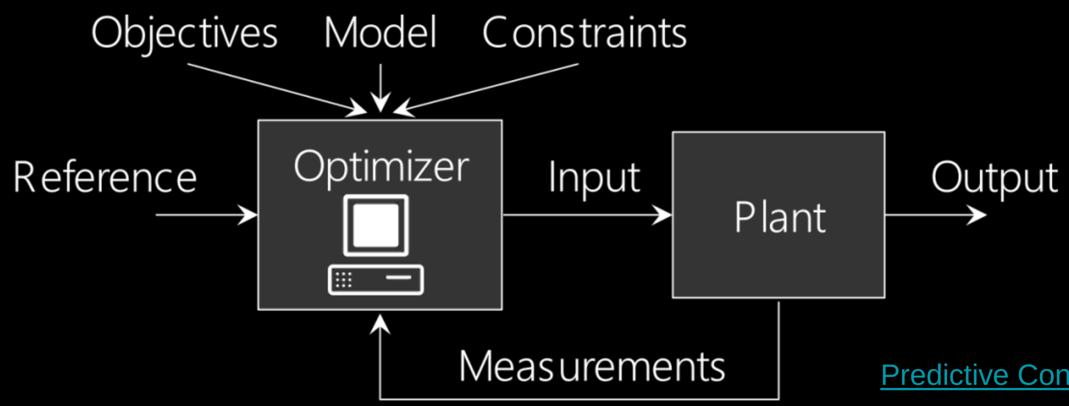
Model predictive control



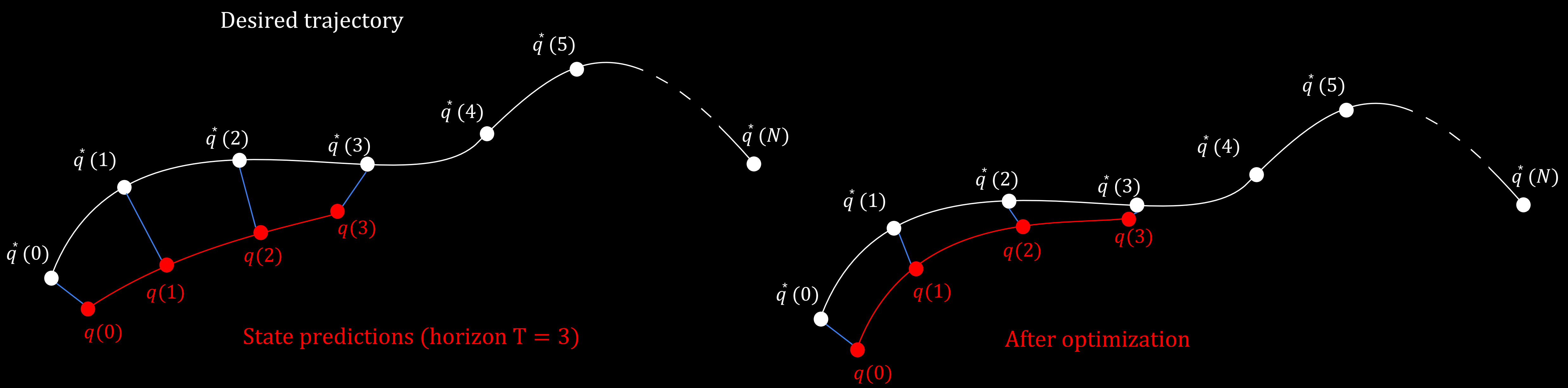
Model Predictive Control (MPC)



[Predictive Control Methods]

- **Idea:** Dynamics model to *predict future states* over a horizon → **optimize** control inputs to minimize a cost.
- **Strengths:** Handles constraints, anticipates the future.
- **Weaknesses:** Computationally expensive, requires an accurate model (sim-to-real gap).

Trajectory following with MPC



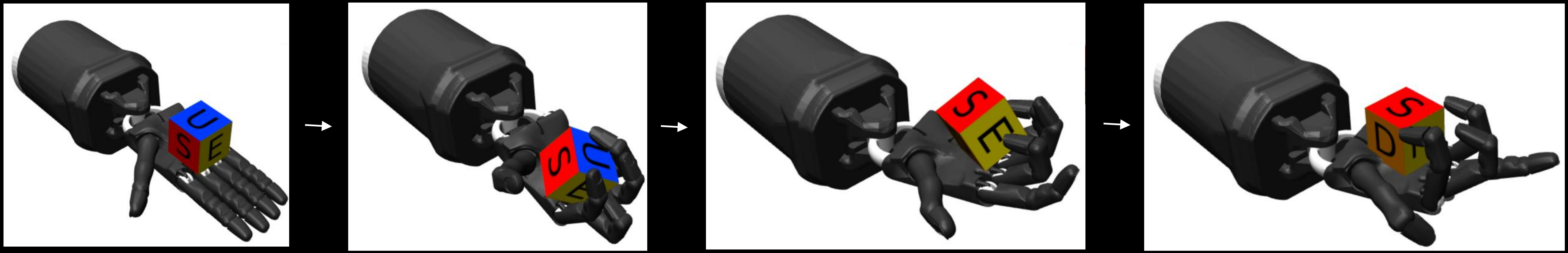
Objective $J = \sum_{t=0}^{T} c(t)$

where each step cost $c(t) = ||q^*(t) - q(t)||_2$

Cube reorientation with MPC

Goal orientation:

Howell et al. 2022, “Predictive Sampling: Real-time Behaviour Synthesis with MuJoCo”



System state $x(t)$ includes robot state $q(t)$, but also the object state.

Objective $J = \sum_{t=0}^{T} c(t)$

where $c(t) = ||cube\ orientation\ (t) - goal\ orientation||_2 + ||cube\ position\ (t) - palm\ center||_2$

Reinforcement Learning (RL) – Overview

Idea: Learn a policy (state → action mapping) by maximizing long-term reward through interaction.

- **Key characteristics:**
  - No explicit model required (model-free RL).
  - Works with nonlinear, high-dimensional dynamics.
- **Strengths:**
  - Captures "intelligent" behaviors.
  - Generalizes beyond what is explicitly modeled.
- **Weaknesses:**
  - Data hungry → needs simulation or many real-world trials (hard and expensive).
  - Transfer from sim to real can be hard (sim-to-real gap).

RL will be covered in **two weeks**.

Feedback Control vs MPC

Feedback control

MPC

Feedback Control vs MPC

Feedback control

- Computationally cheap.

MPC

- Expensive.

Feedback Control vs MPC

Feedback control

- Computationally cheap.
- Reacts to immediate residual.

MPC

- Expensive.
- Longer horizon. But still myopic after horizon T.

Feedback Control vs MPC

Feedback control

- Computationally cheap.
- Reacts to immediate residual.
- Doesn’t require a model.

MPC

- Expensive.
- Longer horizon. But still myopic after horizon T.
- Requires a computational model.
  - Sim2Real gap.

Feedback Control vs MPC

Feedback control

- Computationally cheap.
- Reacts to immediate residual.
- Doesn't require a model.
- Limited to regulation/tracking.

MPC

- Expensive.
- Longer horizon. But still myopic after horizon T.
- Requires a computational model.
  - Sim2Real gap.
- Can encode higher-level tasks.

MPC vs Reinforcement Learning

MPC

Reinforcement Learning

MPC vs Reinforcement Learning

MPC

- No offline training.

Reinforcement Learning

- Offline training needed.

MPC vs Reinforcement Learning

MPC

- No offline training.
- Requires a model.

Reinforcement Learning

- Offline training needed.
- Does not require a model.

MPC vs Reinforcement Learning

MPC

- No offline training.
- Requires a model.
- Limited to our state representations.

Reinforcement Learning

- Offline training needed.
- Does not require a model.
- Can discover latent representations, and "intelligent" behavior.

MPC vs Reinforcement Learning

MPC

- No offline training.
- Requires a model.
- Limited to our state representations.
- Slower during execution.

Reinforcement Learning

- Offline training needed.
- Does not require a model.
- Can discover latent representations, and "intelligent" behavior.
- Learns a policy, a direct mapping from state to action.

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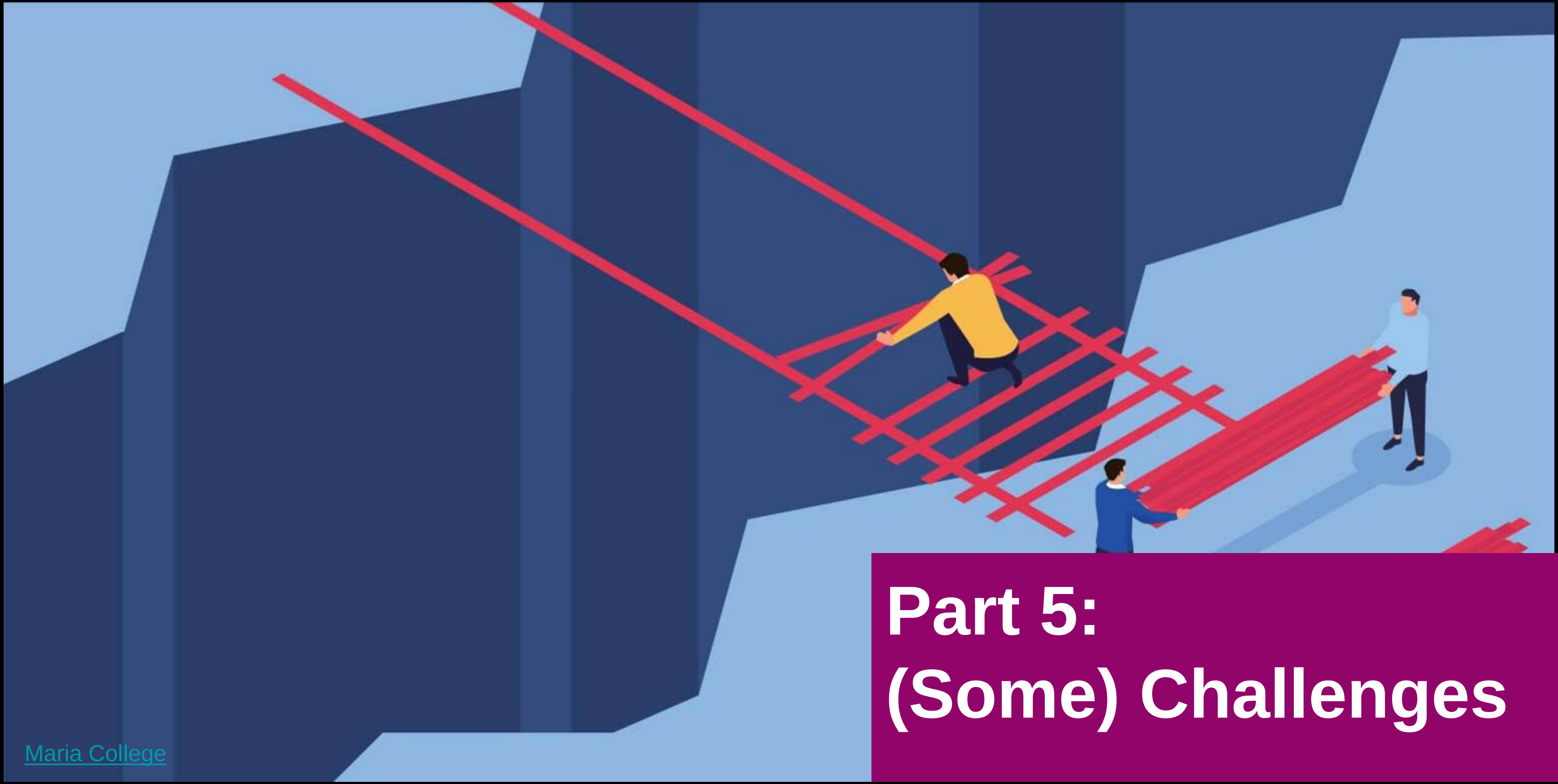

Part 5:
(Some) Challenges

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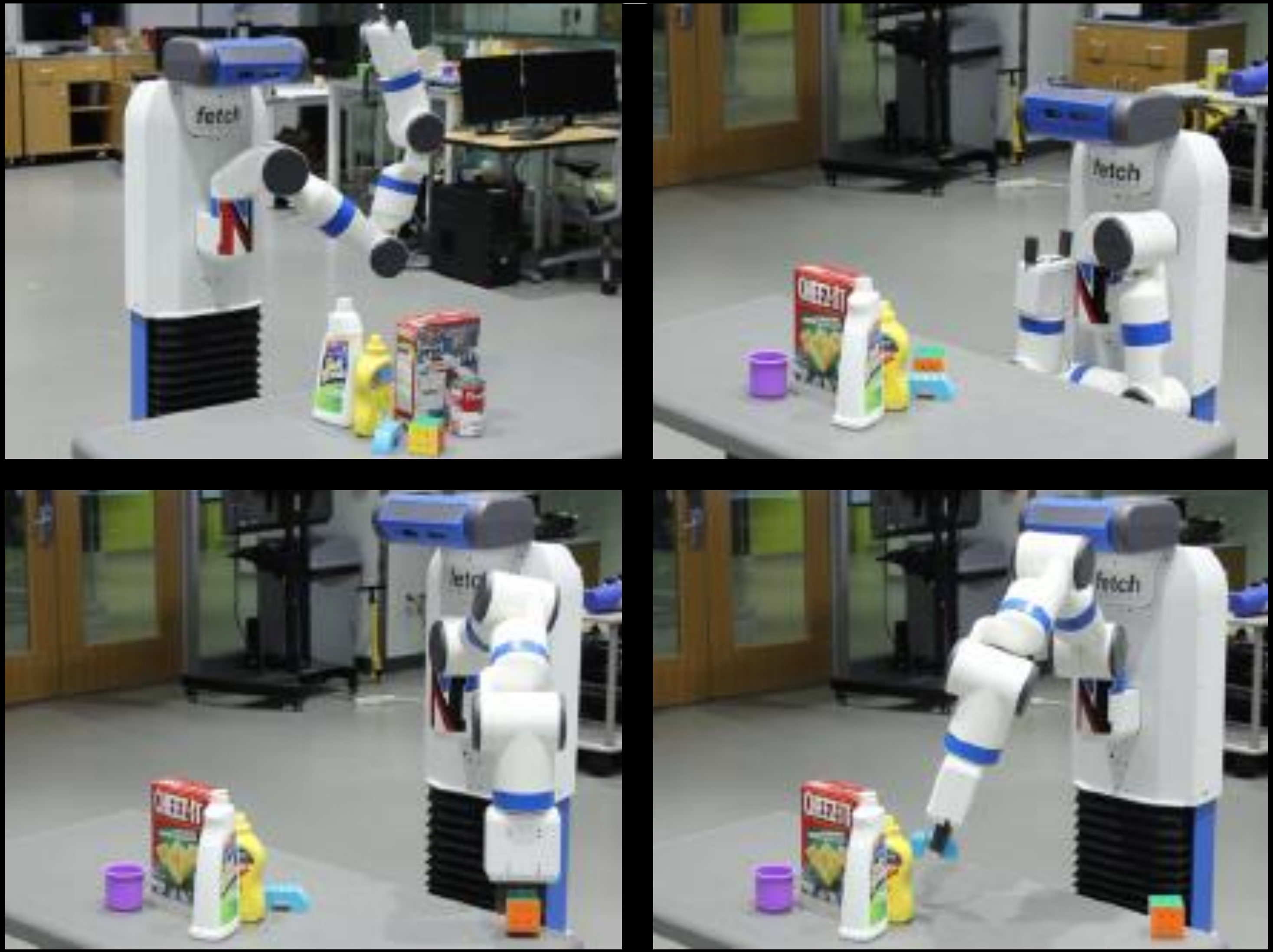
What should you expect?

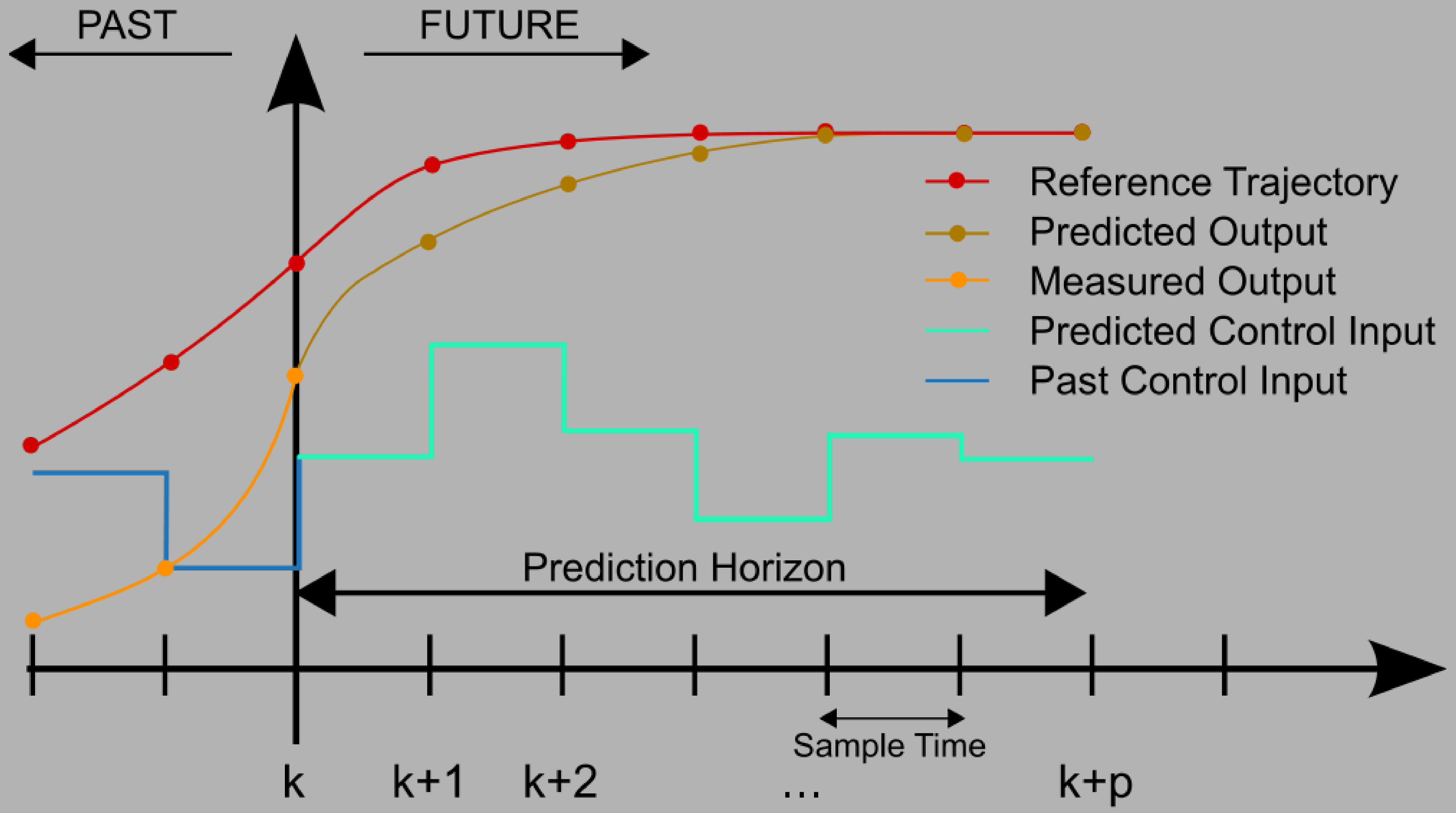
- Uncertainty and Partial Observability
- Long Horizon
- Under/Over actuation
- Sim-to-real gap
- Tendon strain + skin non-linearity
- Encoder's sensibility

Uncertainty and Partial Observability



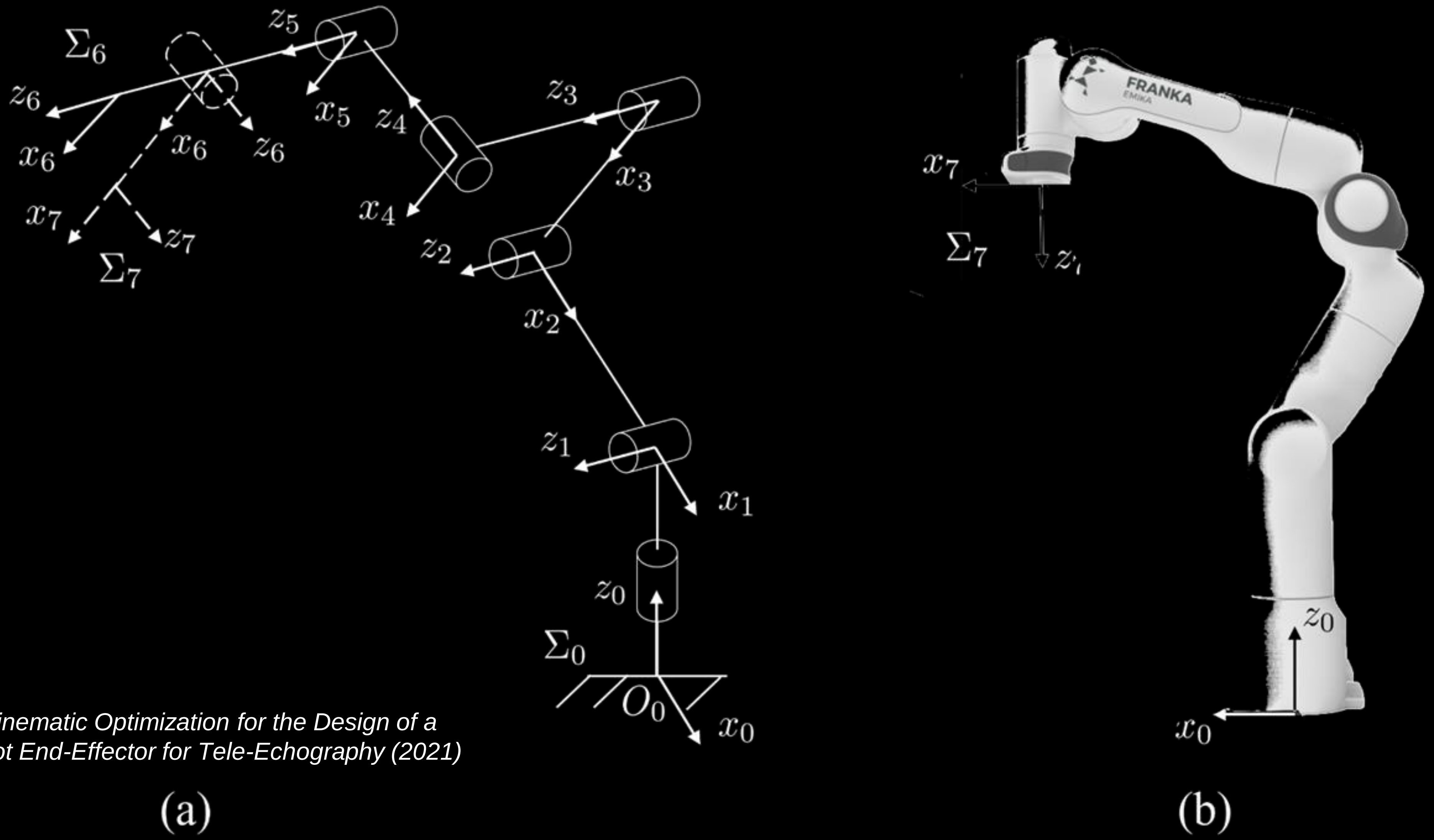
[Yuchen Xiao, Sammie Katt, Andreas ten Pas, Shengjian Chen, Christopher Amato.](#)
[**Online Planning for Target Object Search in Clutter under Partial Observability**.](#)
[IEEE International Conference on Robotics and Automation (ICRA), Montreal, Canada, May 2019.](#)

Long Horizon



[MPC and constrained systems , TU Eindhoven]

Underactuation and Overactuation

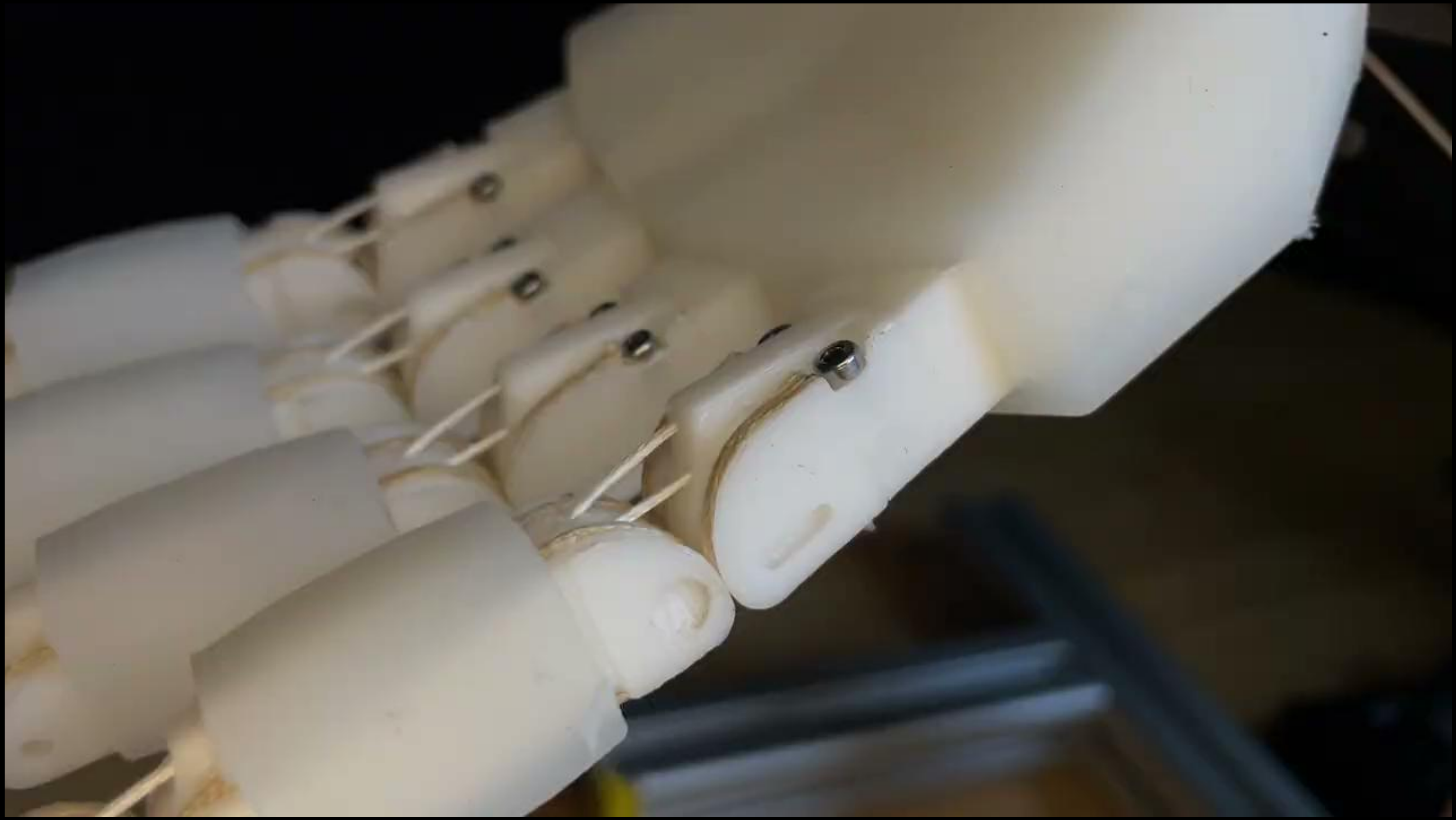


Filippeschi et al. *Kinematic Optimization for the Design of a Collaborative Robot End-Effector for Tele-Echography (2021)*

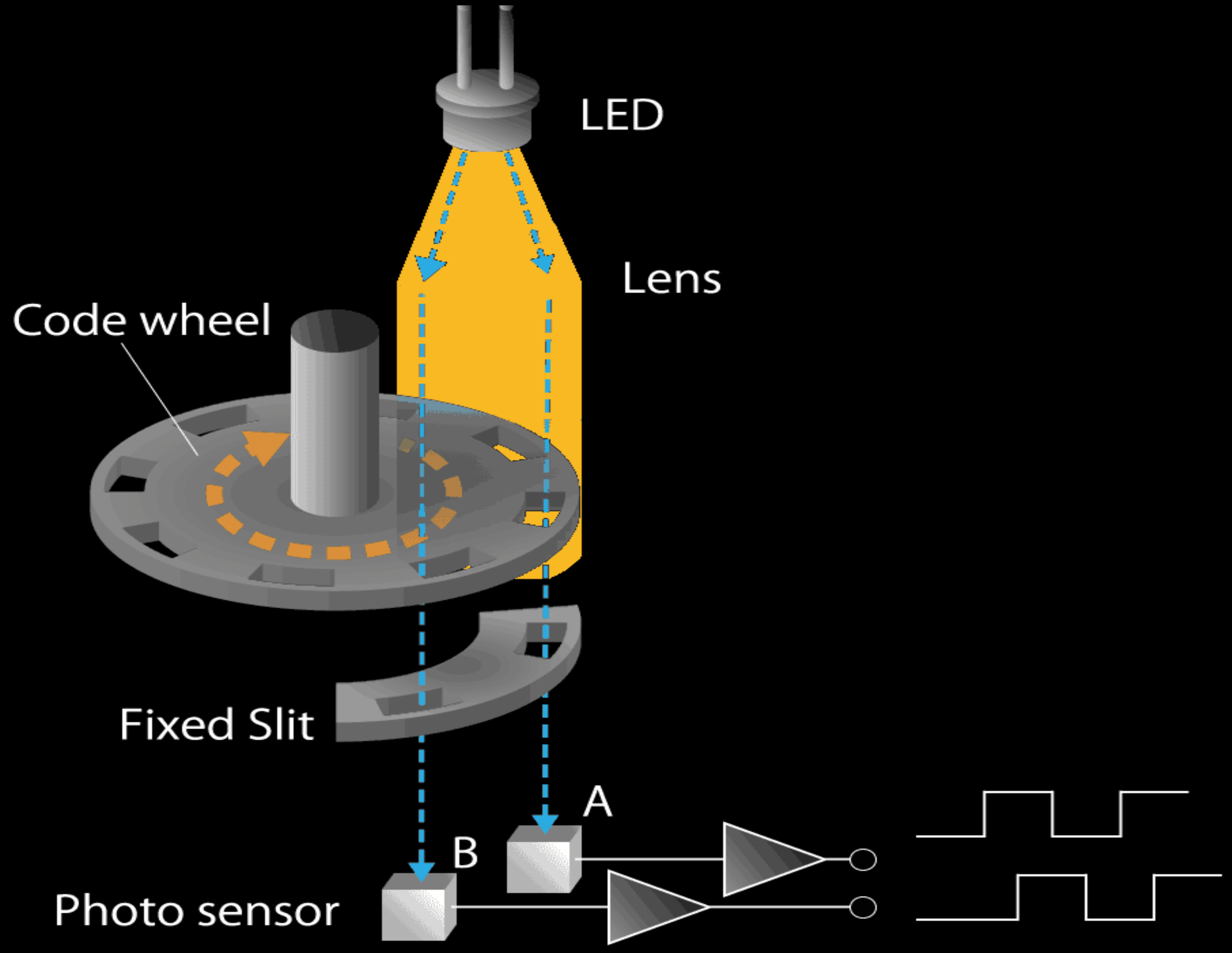
Sim-to-real gap

Everyday Robots

Tendon strain + skin non-linearity



Encoder's sensibility



Asahi Kasei Microdevices

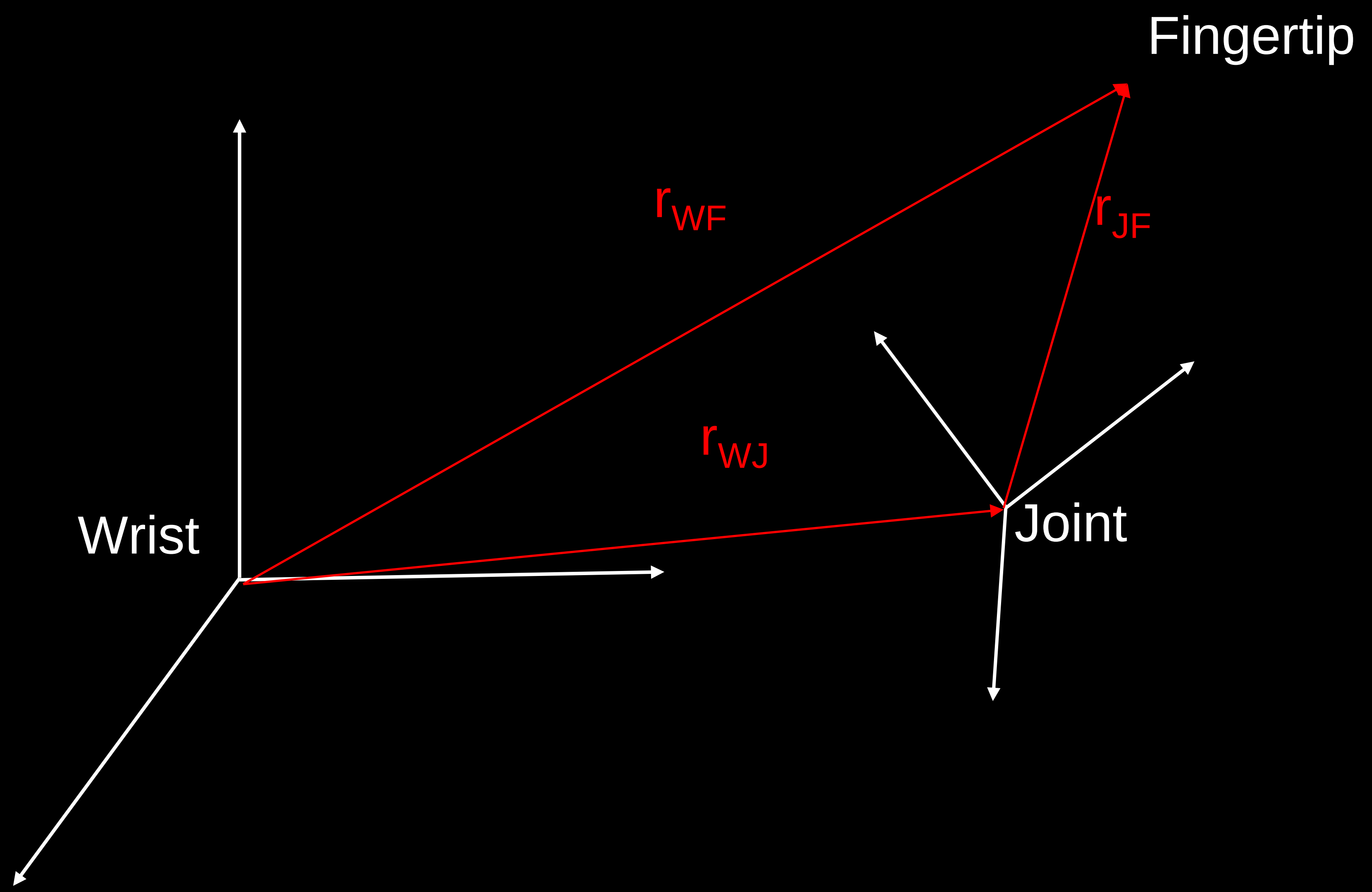
Backup Slides

Sensing the pose: two methods

- Direct methods: Direct reference to the world reference frame
  - The sensors obtain the absolute value of the state we are measuring

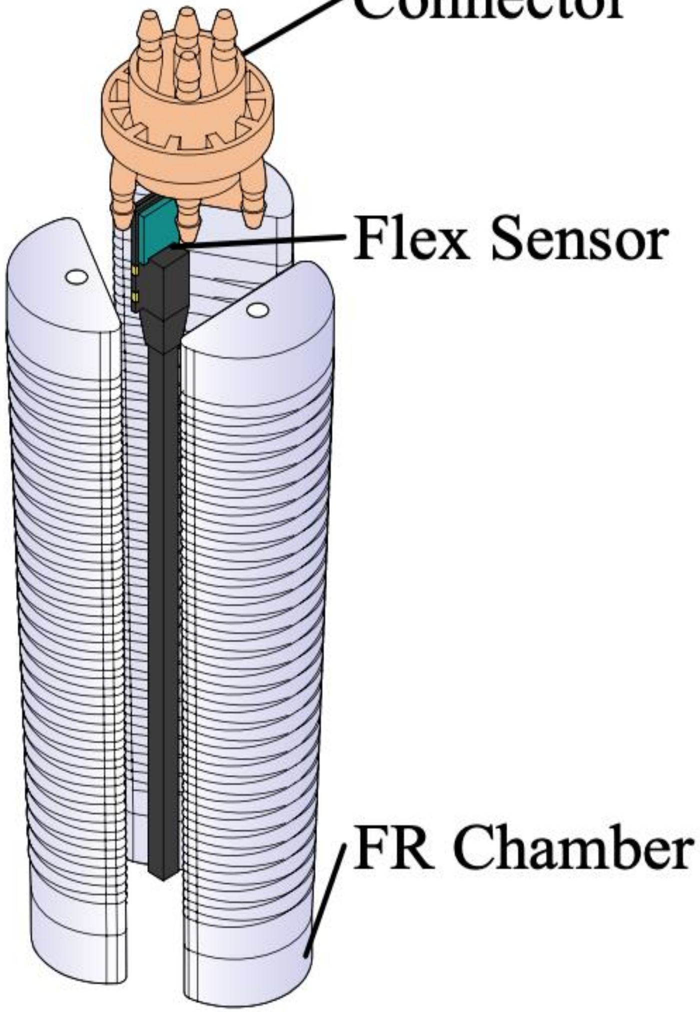
- Indirect methods: Obtain a measurement with reference to a second frame
  - The sensors will estimate a relative measurement that can be transformed into an absolute measurement

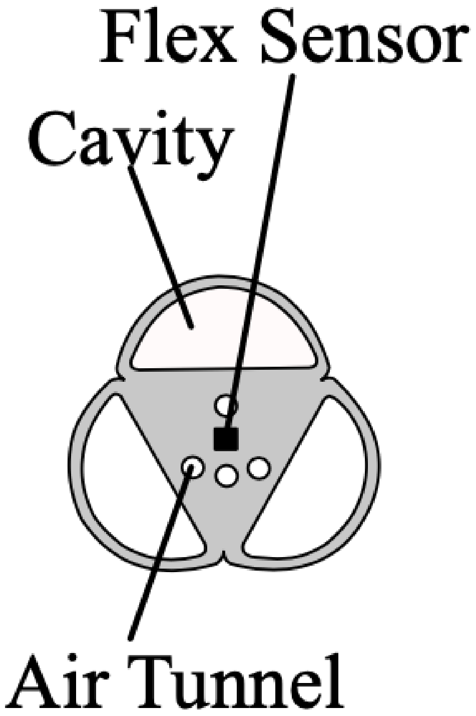
Second solution

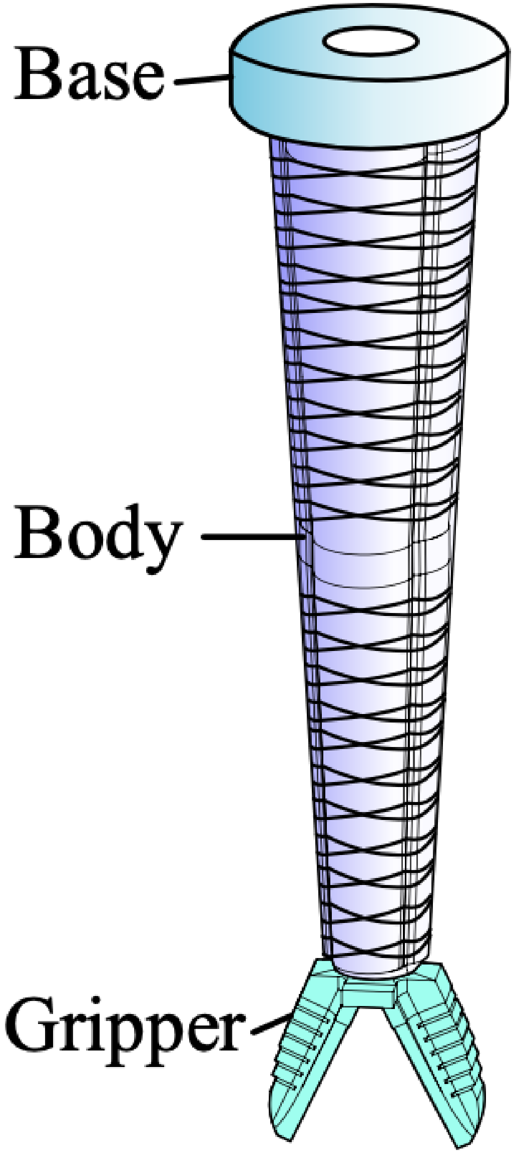


Indirect methods

e.g., Built-in Flex Sensor

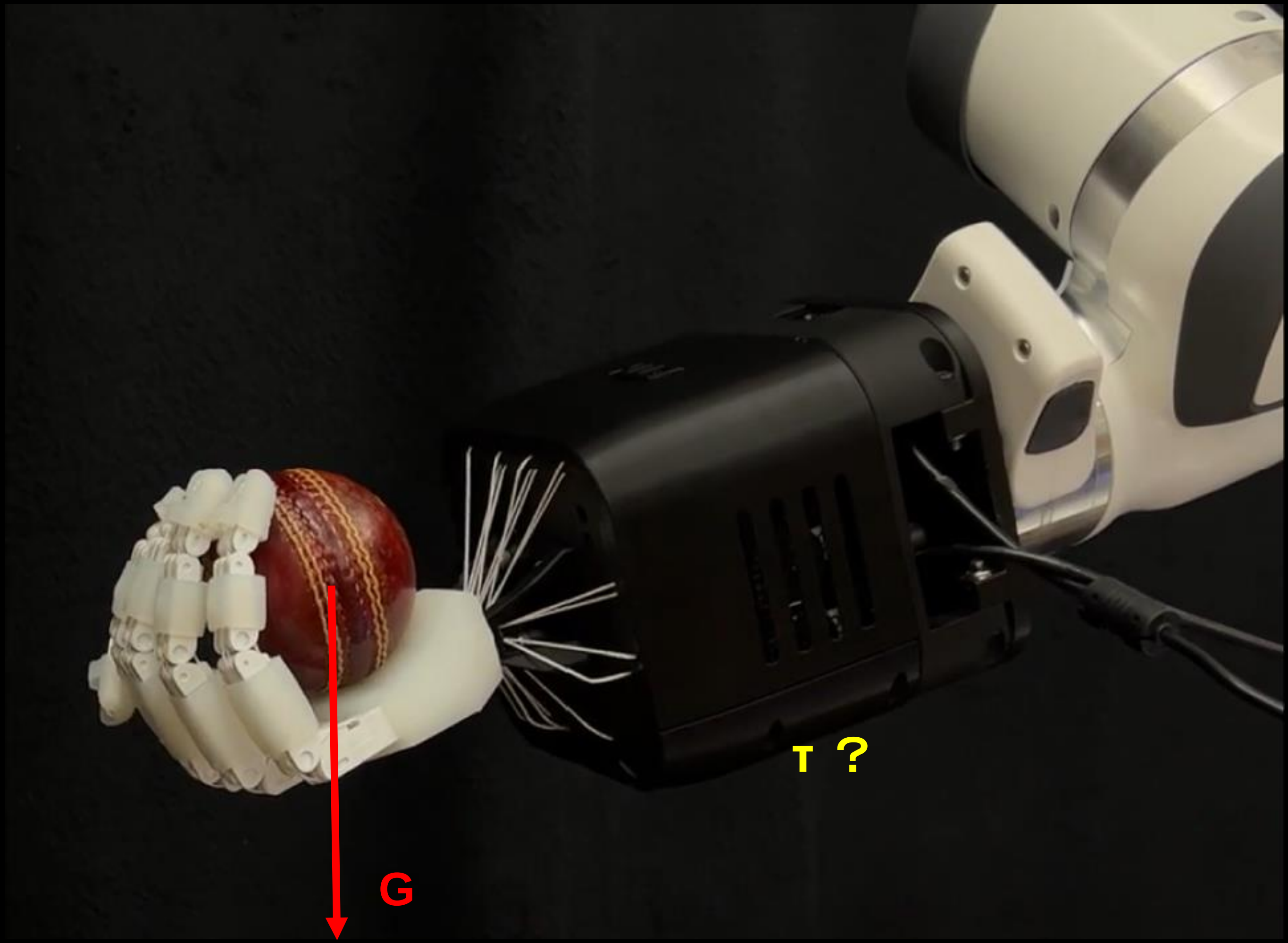






Toshimitsu, Y., Wong, K. W., Buchner, T., & Katzschmann, R. (2021, September). Sopra: Fabrication & dynamical modeling of a scalable soft continuum robotic arm with integrated proprioceptive sensing. In *2021 IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS)* (pp. 653-660). IEEE.

Force control for tendon actuation



Toshimitsu et al. (2023) https://srl-ethz.github.io/get-ball-rolling/

Force control for tendon actuation

Tendon Lengths

$$p = g(l) = g\big(f(q)\big) = F(q)$$

Motor Positions

Joint Angles

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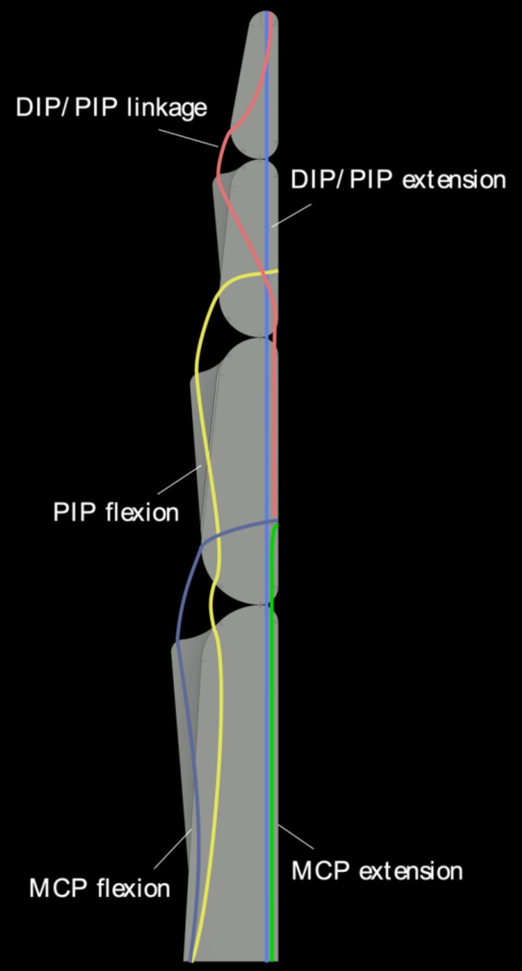
Force control for tendon actuation

$$J_m = \begin{bmatrix} \frac{\partial p_1}{\partial q_1} & \frac{\partial p_1}{\partial q_2} \\ \frac{\partial p_2}{\partial q_1} & \frac{\partial p_2}{\partial q_2} \end{bmatrix}$$



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Force control for tendon actuation

Velocity of the finger joints

$$\dot{p} = J_m \cdot \dot{q}$$

Velocity of the motors

$$\tau^T \cdot \dot{q} = T^T \cdot \dot{p}$$

Conservation of Power

$$\tau^T \cdot \dot{q} = T^T \cdot J_m \cdot \dot{q} \quad \rightarrow \quad \tau = {J_m}^T \cdot T$$

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Force control for tendon actuation

Previous slide: $\tau = {J_m}^T \cdot T$

$$\dot{X}_{fingertip} = J_{fingertip} \cdot \dot{q}$$

$$\tau^T \cdot \dot{q} = {F_{fingertip}}^T \cdot \dot{X}_{fingertip}$$

$$\tau^T \cdot \dot{q} = {F_{fingertip}}^T \cdot J_{fingertip} \cdot \dot{q}$$

$$\tau = {J_{fingertip}}^T \cdot F_{fingertip}$$

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Force control for tendon actuation

$$\tau = {J_m}^T \cdot T$$

$$\tau = {J_{fingertip}}^T \cdot F_{fingertip}$$

$$T = \left({J_m}^T\right)^{-1} \cdot {J_{fingertip}}^T \cdot F_{fingertip}$$

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Outro no slide

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Useful links

https://link.springer.com/book/10.1007/978-3-319-54413-7
https://smartlabai.medium.com/a-brief-overview-of-imitation-learning-8a8a75c44a9c
https://underactuated.csail.mit.edu/index.html
https://www.kalmanfilter.net/default.aspx